Problem Set 8

Assigned: April 9
Due: April 16

Problem 1

Trace the execution of Dijkstra’s algorithm on the following graph, taking A as the starting vertex. Show the successive states of the array $D[i]$ and of the set of vertices whose distance has been determined.

![Graph Diagram]

Problem 2 (Siegel Ex. 8.2)

Write an enhancement to the Floyd-Warshall algorithm that saves, in $\text{Intermediate}[i,j]$, the last vertex before $j$ on the shortest path from $i$ to $j$. Notice that proper initialization makes the algorithm easier.

Problem 3 (Siegel Ex. 8.31)

Suppose that a weighted graph is given with a source vertex and a subset of vertices labelled $S$ (for shop). Give an algorithm to find a shop such that the path from the source to the shop and back again is shortest (the return path need not be the same as the outgoing path).

Hint: This does not require either of the algorithm modification techniques discussed in class. Think about the following problem: given a destination vertex $D$, find all shortest paths from all the other vertices to $D$. 

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Problem 4

Consider the following problem. You are given a directed graph $G$ with two disjoint subsets $A$ and $B$. A path is considered invalid if it goes first through a vertex in $A$ and then through a vertex in $B$. For example $A$ and $B$ may be points in enemy countries, and $B$ may prohibit travellers whose passport has a visa to $A$. Or in an epidemic of a communicable disease, one may want to block people who have been through $A$ from entering $B$.

A. Modify Dijkstra’s algorithm so that it returns the optimal valid paths.

B. Modify the Floyd-Warshall algorithm so that it returns the optimal valid paths.