Problem 1.

Suppose that you are given the problem of returning in sorted order the $k$ smallest elements in an array of size $n$, where $k$ is much smaller than $n$, but much larger than 1.

a. Describe how quicksort can be adapted to this problem. Your description need not give the pseudo-code for the modified algorithms; it is enough simply to describe what changes can be made, as long as your description is clear.

b. In a case where the pivot is always chosen to be the median element, what is the order of magnitude running time of this algorithm, as a function of $k$ and $n$?

Problem 2

We say that an array $A$ is $c$-nice, where $c$ is a constant (e.g., $c = 100$), if for all $1 \leq i, j \leq n$, such that $j - i \geq c$, we have that $A[i] \leq A[j]$. For example, 1-nice array is already sorted. In this problem we will sort such $c$-nice array $A$ using INSERTION SORT and QUICKSORT, and compare the results.

(a) In asymptotic notation (remember, $c$ is a constant) what is the worst-case running time of INSERTION SORT on a $c$-nice array? Be sure to justify your answer.

(b) Suppose that you know that the input array is $c$-nice. How would you choose the pivot for Quicksort? With this choice, what is the what is the worst-case running time? You need not give a rigorous proof of your answer, but give a plausible argument.

(c) Which algorithm would you prefer?

For the running times in parts (a) and (b) all you have to specify is the dependence of the running time on $n$ for a fixed value of $c$; you do not have to say how the running time depends on $c$.

Problem 3.

Prove that any comparison-based method for carrying out the task in problem 1 must take at least time $\Omega(k \cdot \log n)$ in the worst case.

Problem 4.

(Siegel 5.23) Consider the following sorting problem. The input is a sequence of $n$ integers with many duplications, such that the number of distinct integers in the sequence is $O(\log(n))$. Design a sorting algorithm to sort such sequences using at most $O(n \log \log n)$ comparisons in the worst case.