Problem 1

A. Trace the execution of Prim’s algorithm in finding the minimum spanning tree for the graph below. Show the sequence in which edges are added to the tree, and the successive values of the array $D[v]$.

B. Trace the execution of Kruskal’s algorithm in finding the minimum spanning tree for the graph below. Show the sequence in which edges are added to the tree.

\[ \text{Diagram of a graph with nodes A, B, C, D, E, F, G, H, I and edges with weights} \]

Problem 2

(Siegel, Ex. 8.24) Let $G$ be an undirected weighted graph and let $F$ be a subgraph of $G$ that is a forest (a collection of separate trees). Design an efficient algorithm to find a spanning tree of $G$ that contains $F$ and has the minimal total cost over all spanning trees containing $F$.

Problem 3

(Siegel Ex. 8.46) Let $G$ be an undirected connected graph with edge costs, and let $T$ be an MST for $G$. Suppose that due to high winds, some of the edges in $G$ are destroyed. Let $H$ be the edges in $G$ that remain and let $F$ be the edges in $T$ that remain (thus $F = T \cap H$). Assume that $H$ is still connected. Show that there is a minimum spanning tree for $H$ that contains all the edges in $F$. 