Image Measurements and Detection
Images are Intensity Surfaces and Edges are Surface Discontinuities
Measurements: Intensity Accumulation, $\tilde{I}(x, y, \theta, s)$

For each pixel $(x, y)$ we evaluate the accumulation of the intensity along eight (8) directions, $\theta = 0, \pi/4, \pi/2, 3\pi/4, \pi, 5\pi/4, 3\pi/2, 7\pi/4,$ and four (4) different scales, $s = 1, 3, 5, 7$.

$$\tilde{I}(x, y, \theta, s) = \frac{1}{s} \int_0^s I(x + \alpha \cos \theta, y + \alpha \sin \theta) d\alpha$$

Discrete approximation

$$\tilde{I}(x, y, \theta, s) \approx \frac{1}{s} \sum_{i=0}^{s-1} I(x + i \Delta_\theta x \cos \theta, y + i \Delta_\theta y \sin \theta)$$

$\Delta_\theta x = 1$ and $\Delta_\theta y = 1$ for $\theta = 0, \pi/2, \pi, 3\pi/2$

$\Delta_\theta x = \sqrt{2}$ and $\Delta_\theta y = \sqrt{2}$ for $\theta = 0, \pi/4, 3\pi/4, 5\pi/4, 7\pi/4$
Measurements: a Filter “View” of: \( \tilde{I}(x, y, \theta, s) \)

\[
\tilde{I}(x, y, \theta, s) = \frac{1}{s} \int_0^s I(x + \alpha \cos \theta, y + \alpha \sin \theta) \, d\alpha
\]
is a convolution of a normalized filter of constant value \((1/s)\) along a line segment of length \(s\) and angle \(\theta\)

\[
\tilde{I}(x, y, \theta, s) = f \otimes I = \int_0^\infty \int_0^\infty f(x' - x, y' - y, \theta, s) \, I(x', y') \, dx' \, dy'
\]

were

\[
f(X, Y, \theta, s) = \begin{cases} 
\frac{1}{s} & \text{if } (0 \leq \alpha \leq s) \ (X = \alpha \cos \theta \ \&\ \ Y = \alpha \sin \theta) \\
0 & \text{otherwise}
\end{cases}
\]
Measurements: Derivatives $D_\rho \hat{I}(x, y, \theta, s)$

Taking the derivative along the angle $\theta$, i.e., in a direction $\hat{\rho} = (\cos \theta, \sin \theta)$

$$D_\rho \hat{I}(x, y, \theta, s) = \nabla \hat{I}(x, y, \theta, s) \cdot \hat{\rho} = \nabla \hat{I}(x, y, \theta, s) \cdot (\cos \theta, \sin \theta)$$

$$= \frac{\partial \hat{I}(x, y, \theta, s)}{\partial x} \cos \theta + \frac{\partial \hat{I}(x, y, \theta, s)}{\partial y} \sin \theta$$

$$\approx \hat{I}(x, y, \theta, s) - \hat{I}(x - \Delta_\theta x \cos \theta, y - \Delta_\theta y \sin \theta, \pi + \theta, s) .$$

$\Delta_\theta x = 1$ and $\Delta_\theta y = 1$ for $\theta = 0, \pi/2, \pi, 3\pi/2$

$\Delta_\theta x = \sqrt{2}$ and $\Delta_\theta y = \sqrt{2}$ for $\theta = 0, \pi/4, \pi/4, 5\pi/4, 7\pi/4$

We have now a bank of eight (8) distinct oriented filters at different scales, namely

$$\{ D_\rho \hat{I}(x, y, \theta, s) ; \theta = 0, \pi/4, ..., 7\pi/4 \} , \text{and } s = 1, 3, 5, 7, ... \}.$$ 

$| D_\rho \hat{I}(x, y, \theta = \pi/6, s) \}$ will have maximum response (highest value) among the possible orientations, while $| D_\rho \hat{I}(x, y, \theta = \frac{2\pi}{3}, s) \}$ will have the minimum response.
Experiments: $D_\rho \hat{I}(x, y, \theta, s)$

$D_\rho \hat{I}(x, y, 0, 3)$

$D_\rho \hat{I}(x, y, \frac{\pi}{6}, 3)$

$D_\rho \hat{I}(x, y, \frac{\pi}{4}, 3)$

$D_\rho \hat{I}(x, y, \frac{\pi}{3}, 3)$
Exeriments (cont.): $D\hat{I}(x, y, \theta, s)$

$D_{\rho}\hat{I}(x, y, \frac{\pi}{2}, 3)$

$D_{\rho}\hat{I}(x, y, \frac{2\pi}{3}, 3)$

$D_{\rho}\hat{I}(x, y, \frac{3\pi}{4}, 3)$

$D_{\rho}\hat{I}(x, y, \frac{5\pi}{6}, 3)$
Features such as edges, corners, junction, eyes, … are obtained by making some decision from the image measurements.

Decisions are the result of some comparison followed by a choice. Examples (i) if a measurement is above a threshold we accept, not otherwise; (ii) if a measurement is the largest compared to others, we select it.