Numerical Computing Homework 4
Interpolation Assignment
Due: Thursday, March 31.

1. **Written Problem** Find the quadratic interpolant for the data points \((-1, 1), (0, 0), (2, 4),\) using each of the three forms: the monomial basis, the Lagrange basis, and the Newton basis. Show they all give the same quadratic.

2. **Programming Problem** The purpose of this program is to compute an interpolating polynomial for several data sets and see how it behaves.

First, write a Matlab function that takes as input a set of \(n+1\) data points, \((x_i, y_i), i = 0, \ldots, n,\) and computes the coefficients for a cubic polynomial for each interval in the data set. This vector of coefficients should be returned by the function. Since a cubic is determined by 4 points, in the interior intervals you can choose two data points to each side. For the first and last interval you will have to use three points on one side and only one point on the boundary side.

Next write a Matlab function that evaluates the cubic at each point in a vector \(\bar{x}\). This should take as input the \(n+1\) data points, the already determined coefficients, and the vector \(\bar{x}\). To evaluate the cubic use the coefficients from the interval that contains the point. For example, if the point is in the interval \([x_3, x_4]\), then use that cubic (which was defined using the data points at \(x_2, x_3, x_4, x_5\)). You should plot the results of the interpolation at all points \(\bar{x}\), along with the initial \(n+1\) data points.

(a) The first step is for you to make up test data to demonstrate clearly that your program is correct. How did you test your program? What kind of data did you use?

(b) You should run your program on two different data sets of points associated with this homework that are on the class website. Describe the behavior seen in your plots. Can you say something about the respective errors in the two cases?

You should submit your code, your plots, and your discussion, for this part of homework 4.