

Artificial Intelligence – Spring 2007
Assignment 4 solution

8.6

Let the basic vocabulary be as follows:

- $Takes(x, c, s)$: student x takes course c in semester s
- $Passes(x, c, s)$: student x passes course c in semester s
- $Score(x, c, s)$: the score obtained by student x in course c in semester s
- $x > y$: x is greater than y
- F and G : specific French and Greek courses (one could also interpret these sentences as referring to any such course, in which case one could use a predicate $Subject(c, f)$ meaning that the subject of course c is f)
- $Buys(x, y, z)$: x buys y from z (using a binary predicate with unspecified seller is OK but less felicitous)
- $Sells(x, y, z)$: x sells y to z
- $Shaves(x, y)$: person x shaves person y
- $Born(x, c)$: person x is born in country c
- $Parent(x, y)$: x is a parent of y
- $Citizen(x, c, r)$: x is a citizen of country c for reason r
- $Resident(x, c)$: x is a resident of country c
- $Fools(x, y, t)$: person x fools person y at time t
- $Student(x)$, $Person(x)$, $Man(x)$, $Barber(x)$, $Expensive(x)$, $Agent(x)$, $Insured(x)$, $Smart(x)$, $Politician(x)$: predicates satisfied by members of the corresponding categories

- a. $\exists x : Student(x) \wedge Takes(x, F, Spring2001)$
- b. $\forall x, s : Student(x) \wedge Takes(x, F, s) \Rightarrow Passes(x, F, s)$
- c. $\exists x : Student(x) \wedge Takes(x, G, Spring2001) \wedge \forall y : (y \neq x) \Rightarrow \neg Takes(y, G, Spring2001)$
- d. $\forall s \exists x \forall y : Score(x, G, s) > Score(y, F, s)$
- e. $\forall x : Person(x) \wedge (\exists y, z : Policy(y) \wedge Buys(x, y, z)) \Rightarrow Smart(x)$
- f. $\forall x, y, z : Person(x) \wedge Policy(y) \wedge Expensive(y) \Rightarrow \neg Buys(x, y, z)$
- g. $\exists x : Agent(x) \wedge \forall y, z : Policy(y) \wedge Sells(x, y, z) \Rightarrow (Person(z) \wedge \neg Insured(z))$
- h. $\exists x : Barber(x) \wedge \forall y : Man(y) \wedge \neg Shaves(y, y) \Rightarrow Shaves(x, y)$
- i. $\forall x : Person(x) \wedge Born(x, UK) \wedge (\forall y : Parent(y, x) \Rightarrow ((\exists r : Citizen(y, UK, r)) \vee Resident(y, UK)))$
 $\Rightarrow Citizen(x, UK, Birth)$
- j. $\forall x : Person(x) \wedge \neg Born(x, UK) \wedge (\exists y : Parent(y, x) \wedge Citizen(x, UK, Birth)) \Rightarrow Citizen(x, UK, Descent)$

- $\forall x : \text{Politician}(x) \Rightarrow$
- k. $(\exists y \forall t : \text{Person}(y) \wedge \text{Fools}(x, y, t)) \wedge$
 $(\exists t \forall y : \text{Person}(y) \Rightarrow \text{Fools}(x, y, t)) \wedge$
 $\neg(\forall t \forall y : \text{Person}(y) \Rightarrow \text{Fools}(x, y, t))$

9.4

- $\{x/A, y/B, z/B\}$ (or some permutation of this)
- No unifier (x cannot bind to both A and B)
- $\{y/\text{John}, x/\text{John}\}$
- No unifier (because the occurs-check prevents unification of y with $\text{Father}(y)$)

9.6

From We will give the average-case time complexity for each query/scheme combination in the following table (an entry of the form “1; n ” means that it is $O(1)$ to find the first solution to the query, but $O(n)$ to find them all). We make the following assumptions: hash tables give $O(1)$ access; there are n people in the data base; there are $O(n)$ people of any specific age; every person has one mother; there are H people in Houston and T people in Tiny Town; T is much less than n ; in Q4, the second conjunct is evaluated first.

	Q1	Q2	Q3	Q4
S1	1	1; H	1; n	$T; T$
S2	1	$n; n$	1; n	$n; n$
S3	n	$n; n$	1; n	$n^2; n^2$
S4	1	$n; n$	1; n	$n; n$
S5	1	1; H	1; n	$T; T$

Anything that is $O(1)$ can be considered “efficient”, as perhaps can anything $O(T)$. Note that S1 and S5 dominate the other schemes for this set of queries. Also note that indexing on predicates plays no role in this table (except in combination with an argument). Because there are only 3 predicates (which is $O(1)$). It would make a difference in terms of the constant factor.

9.19

- (A) translates to “For every natural number there is some other natural number that is smaller than or equal to it”. (B) translates to “There is a particular natural number that is smaller than or equal to any natural number”.
- Yes, (A) is true under this interpretation. You can always pick the number itself for the “some other” number.
- Yes, (B) is true under this interpretation. You can pick 0 for the “particular natural number”.
- No, (A) does not logically entail (B).
- Yes, (B) logically entails (A).
- We want to try to prove via resolution that (B) entails (A). To do this, we set our knowledge base to consist of (B) and the negation of (A), which we will call $(\neg A)$, and

try to derive a contradiction. First we have to convert (-A) and (B) to canonical form. For (-A), this involves moving the \neg in past the two quantifiers. For both sentences, it involves introducing a Skolem function:

- (-A) $\neg F_1() \geq y$
- (B) $x \geq F_2()$

The resolution goes through, with the substitution $\{x/F_1, y/F_2\}$, thereby yielding *False*, and proving that (B) entails (A).

g. To prove that (A) entails (B), we start with a knowledge base containing (A) and the negation of (B), which we will call (-B):

- (A) $x \geq F_1(x)$
- (-B) $\neg F_2(y) \geq y$

Now we can try to resolve these two together, but the occurs check rules out the unification. It looks like the substitution should be $\{x/F_2(y), y/F_1(x)\}$, but that is equivalent to $\{x/F_2(y), y/F_1(F_2(y))\}$, which fails because y is bound to an expression containing y . So the resolution fails, there are no other resolution steps, and therefore (B) does not follow from (A).