Homework #2: Symmetry Axis

Due October 7th, 2015
Professor Davi Geiger.

The goal of this assignment is to learn about Symmetry axis.

1. Basic Code: Symmetry Axis Detection on Simple Images

   a. Parallel Lines
   b. Varying width, same axis.

   Let us work with $\sigma=3$.

   $\theta = \phi - \pi/2 + \Delta \theta$

   Stencil
Basic algorithm to construct the Space of all votes.

1. Compute Wavelet responses, $J(x,y,\theta)$, for 16 directions from 0 to $\pi$:
   \[ \theta=0, \pi/16, \pi/8, ..., 7\pi/8, 15\pi/16. \]
2. $\text{Sym}(\rho, \phi, d, cx, cy) = 0$; % initialize, this is the Symmetry Accumulator
3. for $\phi$ (say 16 directions from 0 to $\pi$): $\phi=0, \pi/16, \pi/8, ..., 7\pi/8, 15\pi/16$.
4. for $c$ (all centers, points, in the image, $c=(cx,cy)$)
   % in matlab this loop can be done as one simultaneous operation (see class material)
5. $\rho = cx \cos (\phi - \pi/2) + cy \sin (\phi - \pi/2)$; % Hough Space of lines
6. end
7. for $\Delta \theta = -\pi/4, 0, \pi/4$ % 3 angle responses only
8. $\theta = \phi - \pi/2 + \Delta \theta$; % Notice that $\theta \in [0, \pi/16, \pi/8, ..., 7\pi/8, 15\pi/16]$
   Thus, if needed, add or subtract $\pi$.
9. $\theta' = 2\phi - \theta$; % Same here, if needed add or subtract $\pi$.
10. for all $d < \text{half of the image diagonal}$
11. $(x, y) = c - d (\cos (\phi - \pi/2), \sin (\phi - \pi/2))$
12. $(x', y') = c + d (\cos (\phi - \pi/2), \sin (\phi - \pi/2))$
13. $\text{MS}(c, d, \phi) = J(x,y,\theta) J*(x',y',\theta')$;
14. $\text{Sym}(\rho, \phi, d, cx, cy) = \text{Sym}(\rho, \phi, d, cx, cy) + \text{MS}(c, d, \phi)$;
15. end
16. end
17. end
18. end
19. Return($|\text{Sym}(\rho, \phi, d, cx, cy)|^2 = \text{Sym}(\rho, \phi, d, cx, cy) \text{Sym}^*(\rho, \phi, d, cx, cy)$);
   % multiplication by the conjugate
2. Can we find the range “d”: $d_{\text{min}}$ and $d_{\text{max}}$?

Consider now the histogram

$$H(d)=|\text{Sym}(d)|^2 = \sum_{\rho, \phi, cx, cy} |\text{Sym}(\rho, \phi, d, cx, cy)|^2$$

Show the histogram $H(d)=|\text{Sym}(d)|^2$ or better/smooth $H(d)=\text{Gaussian}*|\text{Sym}(d)|^2$.

We now expect the range ($d_{\text{min}}, d_{\text{max}}$) to be found from the Histogram $H(d)$.

a. Convert $H(d)$ into $\text{Prob}(d)$, by normalizing to 1, i.e., $\text{Prob}(d) = \frac{H(d)}{\sum d H(d)}$

b. Compute $d_{\text{best}} = \text{argmax} \text{Prob}(d)$ (the one that maximize the probability).

c. Find $d_{\text{min}}$ and $d_{\text{max}}$ as the first neighbors of $d_{\text{best}}$ such that $\text{Prob}(d)=0.5*\text{Prob}(d_{\text{best}})$

3. Find the symmetry axis

In order to avoid clutter data, use $d_{\text{min}}$ and $d_{\text{max}}$ from previous exercise as the range of $d$ and consider now the histogram

$$H(\rho, \phi)=|\text{Sym}(\rho, \phi)|^2 = \sum_{d, cx, cy} |\text{Sym}(\rho, \phi, d, cx, cy)|^2$$

Show the histogram $H(\rho, \phi)$ as a picture and extract the maximum $(\rho, \phi)^*$, and show the line correspondent to these parameters $(\rho, \phi)^*$, overlaid on the image.

Use now your own data set. Marcelo will be putting together the whole data set for everyone to use it, but this will happen later, after next class. Well, if your images are very large, you may want to resize them to small images, where the longer length is 200 pixels? In this way these computations will not take too long to run. Use any technique to resize, but make sure that the resulting image still shows the intended symmetry.

4. Run the program developed in 2, on your own data set. Show the histograms $H(d)$. Does it still has peaks at the reasonable values of “d” (half distances of the symmetric objects) for each image?

5. Run the program developed in 3, using reasonable ranges of $d$ (for each image), either from the results of 3. Or if they are not reasonable, put it by hand the range, which must include the answers you expect. Show results.