GRANOVETTER’S WEAK AND STRONG TIES: A SIMULATION

Towards Building a Social Network Graph:

**Strong ties: Problems 1, 2, 3, 4.**

We will specify a clique with $N$ nodes. We will specify the initial probability $\mathbf{p}^0 = (p_i^0, ..., p_{i=N}^0)$ of each node to accept an innovation. We will compute $\mathbf{p}^t = (p_i^t, ..., p_{i=N}^t)$. We will specify the strong ties parameters $a, b$ for the entire clique. Consider the following update routines

**UpdateStrongTie** ($p_i^t, a_{ij}, b_{ij}$)

create $p_{ij}^{t+1} = (p_{ij}^{t+1}(s = 1), p_{ij}^{t+1}(s = 0))$;

$p_{ij}^{t+1}(s_i = 1) = a_{ij} p_j^t(s_j = 1) + (1 - b_{ij}) p_i^t(s_j = 0)$;

$p_{ij}^{t+1}(s_i = 0) = (1 - a_{ij}) p_j^t(s_j = 1) + b_{ij} p_i^t(s_j = 0)$;

return $p_{ij}^{t+1}$

**UpdateStrongTies-to-i** ($\mathbf{p}^t, a, b, i, N$)

/* implements $p_{ij}^{t+1}(s_i = 1) = \prod_{j=1}^N \left( a_{ij} p_j^t(s_j = 1) + (1 - b_{ij}) p_i^t(s_j = 0) \right)$;

and $p_{ij}^{t+1}(s_i = 0) = \prod_{j=1}^N \left( (1 - a_{ij}) p_j^t(s_j = 1) + b_{ij} p_i^t(s_j = 0) \right)$; */

$p_i^{t+1} = 1$; /* note this is a vector: $(p_i^{t+1}(s_i = 1), p_i^{t+1}(s_i = 0)) = (1, 1)$ */

loop for $j=2,...,N$

$p_{ij}^{t+1} = $UpdateStrongTie ($p_i^t, a_{ij}, b_{ij}$);

$p_i^{t+1}(s_i = 1) = (p_i^{t+1}(s_i = 1) \times p_{ij}^{t+1}(s_i = 1))$;

$p_i^{t+1}(s_i = 0) = (p_i^{t+1}(s_i = 0) \times p_{ij}^{t+1}(s_i = 0))$;

end loop

$C_i = p_i^{t+1}(s_i = 1) + p_i^{t+1}(s_i = 0)$; /* normalization */

$p_i^{t+1}(s_i) = \left( \frac{p_i^{t+1}(s_i = 1)}{C_i}, \frac{p_i^{t+1}(s_i = 0)}{C_i} \right)$; /* normalized updated probability */

return $p_i^{t+1}$
UpdateStrongTies \( (p^t, a, b, N) \)

\[
\text{loop for } i=1,\ldots,N \\
p_{i}^{t+1} = \text{UpdateStrongTies-to-}i \ (p^t, a, b, i, N); \\
\text{end loop} \\
\text{return } p^{t+1} = (p_{i=1}^{t+1}, \ldots, p_{i=N}^{t+1}) \\
\]

StrongTies \( (p^t=0, a, b, T) \)

\[
\text{loop for } t=1,\ldots,T \\
p^t = \text{StrongTies} \ (p^{t-1}, a, b); \\
\text{end loop} \\
\text{return } p^t \\
\]

A stable solution occurs when \( p^{t+1} = p^t \rightarrow (p_{i=1}^{t+1}, \ldots, p_{i=N}^{t+1}) = (p_{i=1}^t, \ldots, p_{i=N}^t) \).

Typically we look for solutions that are stable with an error \( \epsilon \). We mean the maximum error is

\[
p_{i}^{t+1}(s_i = 1) - p_{i}^t(s_i = 1) \leq \epsilon \quad i = 1, \ldots, N
\]

Note that the error for state \( s_i = 0 \), \( p_{i}^{t+1}(s_i = 0) - p_{i}^t(s_i = 0) \) is the same as \( p_{i}^{t+1}(s_i = 1) - p_{i}^t(s_i = 1) \). In our case let us choose \( \epsilon = 0.01 \).

Specify \( N=4 \),

\[
a = b = \begin{pmatrix}
0.8, & 0.6, & 0.7, & 0.6 \\
0.5, & 0.7, & 0.6, & 0.5 \\
0.6, & 0.7, & 0.6, & 0.6 \\
0.5, & 0.6, & 0.7, & 0.5 \\
\end{pmatrix}
\]

or \( a = \begin{pmatrix}
a_{11} = 0.8, a_{12} = 0.6, a_{13} = 0.7, a_{14} = 0.6 \\
a_{21} = 0.5, a_{22} = 0.7, a_{23} = 0.6, a_{24} = 0.5 \\
a_{31} = 0.6, a_{32} = 0.7, a_{33} = 0.6, a_{34} = 0.6 \\
a_{41} = 0.5, a_{42} = 0.6, a_{43} = 0.7, a_{44} = 0.5 \\
\end{pmatrix} = b,
\]

\[
p^0(s) = \{p_1^0 = (0.4, 0.6), p_2^0 = (0.5, 0.5), p_3^0 = (0.7, 0.3), p_4^0 = (0.8, 0.2)\};
\]

1. Compute, for \( i=1 \) (first node) and for \( i=3 \) (third node), the function \( p_{i}^{t=1} = \text{UpdateStrongTies-to-}i \ (p^0, a, b, i, N) \);

Note that the input \( p^0, a, b, N \) are all given above.

2. Compute, \( p^T \). Apply \( p^T = \text{StrongTies} \ (p^t=0, a, b, T) \) for increasing values of \( T \) to see how long it takes to reach a stable value. Report on the minimum value \( T = T_{\text{min}} \), to reach stable solution and make four plots, one for each \( p_{i}^{t=1}(s_i = 1) \) over time (i.e, x-axis is time \( t=1,\ldots,T_{\text{min}} \) and y-axis is \( p_{i}^{t=1}(s_i = 1) \).
3. Robustness: add a different random variable between 0 and 0.01, to each element of \( a \) (still \( b = a \)). Print \( a \). Repeat item 2. Is it robust?

4. Consider now \( \mathbf{p}^0 = \{ p^0_1 = (0.7, 0.3), p^0_2 = (0.5, 0.5), p^0_3 = (0.2, 0.8), p^0_4 = (0.4, 0.6) \} \);

apply \( \mathbf{p}^T = \text{StrongTies} (\mathbf{p}^{t=0}, a, b, T) \) for increasing values of \( T \) to see how long it takes to reach a stable value. Report on the minimum value \( T = T_{min} \), to reach stable solution and make four plots, one for each \( p_i^{t=1}(s_i = 1) \) over time (i.e, x-axis is time \( t=1, \ldots, T_{min} \) and y-axis is \( p_i^{t=1}(s_i = 1) \)).

**Weak ties: Problems 5 & 6.**

We will specify a node \( j \) to pass an innovation and a clique node \( i \) to accept a weak tie.

We will specify the initial probabilities \( \mathbf{p}^0 = (p_j^0, p_i^0) \) of each node to accept an innovation. We will compute \( \mathbf{p}^t = (p_j^t, p_i^t) \)

We will specify the weak ties parameters \( \alpha, \beta \)

Consider the following update routines

\[
\text{UpdateWeakTies} (\mathbf{p}^t, \alpha_{ij}, \beta_{ij}, a_{ii}, b_{ii})
\]

\[
p_i^{t+1}(s_i = 1) = \left( \alpha_{ij} p_j^{t}(s_j = 1) + (1 - \beta_{ij}) p_j^{t}(s_j = 0) \right) \\
\times \left( a_{ii} p_i^{t}(s_i = 1) + (1 - b_{ii}) p_i^{t}(s_i = 0) \right);
\]

\[
p_i^{t+1}(s_i = 0) = \left( (1 - \alpha_{ij}) p_j^{t}(s_j = 1) + \beta_{ij} p_j^{t}(s_j = 0) \right) \\
\times \left( (1 - a_{ii}) p_i^{t}(s_i = 1) + b_{ii} p_i^{t}(s_i = 0) \right)
\]

\[
C_i = p_i^{t+1}(s_i = 1) + p_i^{t+1}(s_i = 0);
\]

\[
p_i^{t+1}(s_i) = \left( \frac{p_i^{t+1}(s_i = 1)}{C_i}, \frac{p_i^{t+1}(s_i = 0)}{C_i} \right);
\]

end loop

return \( \mathbf{p}^{t+1} = (p_i^{t+1}(s_i), p_j^{t}(s_j)) \)
Note that we also return the value $p_j^t(s_j)$ unchanged.

**WeakTies** ($p^{t=0}, \alpha_{ij}, \beta_{ij}, a_{ii}, b_{ii}, T$)

loop for $t=1, \ldots, T$

$p^t = $UpdateWeakTies ($p^{t-1}, \alpha_{ij}, \beta_{ij}, a_{ii}, b_{ii}$);

end loop

return $p_{i}^{t+1}(s_i)$

Note that here we only return $p_{i}^{t+1}(s_i)$

**Problems 5 & 6.**

1. Consider $N=4$, $a_{ii} = 0.6 = b_{ii}$, $\alpha_{ij} = \beta_{ij} = 0.7$

   $p^0 = \{p_i^0(s) = (0.4, 0.6), p_j^0(s) = (0.7, 0.3)\}$;

   apply $p_i^T(s_i) =$ WeakTies ($p^{t=0}, \alpha_{ij}, \beta_{ij}, a_{ii}, b_{ii}, T$) for increasing values of $T$ to see how long it takes to reach a stable value (here we examine only $p_i^T(s_i)$). Report on the minimum value $T = T_{\text{min}}$, to reach stable solution and make four plots, one for each $p_i^t(s_i = 1)$ over time (i.e, x-axis is time $t=1, \ldots, T_{\text{min}}$ and y-axis is $p_i^t(s_i = 1)$).

2. Consider the same $N=4$, $a_{ii} = 0.6 = b_{ii}$ but modify $\alpha_{ij} = \beta_{ij} = 0.5$ again

   apply $p_i^T(s_i) =$ WeakTies ($p^{t=0}, \alpha_{ij}, \beta_{ij}, a_{ii}, b_{ii}, T$) for increasing values of $T$ to see how long it takes to reach a stable value. Report on the minimum value $T = T_{\text{min}}$, to reach stable solution and make four plots, one for each $p_i^t(s_i = 1)$ over time (i.e, x-axis is time $t=1, \ldots, T_{\text{min}}$ and y-axis is $p_i^t(s_i = 1)$).