Exercises 5

Assigned: Nov. 4
Due: Nov. 18

Exercise 1

(You may use MATLAB).

A patient comes into a doctors office exhibiting two symptoms: s1 and s2. The doctor has two possible diagnoses: disease d1 or disease d2. Assume that, given the symptoms, the patient must have either d1 or d2, and cannot have both. We are given the following probabilities:

\[ P(s1 \mid d1) = 0.8. \]
\[ P(s1 \mid d2) = 0.4. \]
\[ P(s2 \mid d1) = 0.2. \]
\[ P(s2 \mid d2) = 0.6. \]
\[ P(d1) = 0.003. \]
\[ P(d2) = 0.007. \]

Assume that s1 and s2 are conditionally independent given the disease.

A. What are \( P(d1 \mid s1,s2) \) and \( P(d2 \mid s1,s2) \)?

B. The doctor has the choice of two treatments, t1 and t2. (It is not an option to do both.) Let c be the event that the patient is cured. We are given the following probabilities:

\[ P(c \mid d1,t1) = 0.8 \]
\[ P(c \mid d2,t1) = 0.1 \]
\[ P(c \mid d1,t2) = 0.3 \]
\[ P(c \mid d2,t2) = 0.6 \]

Assume that the event c is conditionally independent of the symptoms, given the disease and the treatment. What is \( P(c \mid t1,s1,s2) \)? What is \( P(c \mid t2,s1,s2) \)?

C. Suppose that treatment t1 has cost $1000 and treatment t2 has cost $500. If the patient has disease d1, then the value of being cured is $20,000; if the patient has disease d2, then the value of being cured is $15,000. Given that the patient is exhibiting symptoms s1 and s2, what is the expected value of applying t1? What is the expected value of applying t2?

D. The doctor also has the option of ordering a test with a Boolean outcome. The test costs $800. Logically, tests are like symptoms, so let event s3 be a positive result on this test. We are given the following probabilities:

\[ P(s3 \mid d1) = 0.9 \]
\[ P(s3 \mid d2) = 0.1 \]

Assume that s3 is conditionally independent of s1 and s2 given the disease. Is it worthwhile ordering the test? What is the expected gain/cost from ordering the test?
Exercise 2

(You may use MATLAB)

A publisher needs to decide whether to publish a book that has been submitted. Let us simplify the situation by assuming that, if the book is published, there are two possible outcomes: Success and Failure. If Success then the publisher will make $50,000; if Failure, then the publisher will lose $10,000. If the publisher doesn’t publish the book, then he breaks even. The publisher can hire a reviewer for amount $500. The reviewer gives two possible reviews: For or Against. We are given the following probabilities:

\[
\begin{align*}
P(\text{Success}) &= 0.2 & P(\text{Failure}) &= 0.8 \\
P(\text{For} | \text{Success}) &= 0.7 & P(\text{Against} | \text{Success}) &= 0.3 \\
P(\text{For} | \text{Failure}) &= 0.4 & P(\text{Against} | \text{Failure}) &= 0.6
\end{align*}
\]

A. What is the publisher’s expected gain from consulting the reviewer?

B. The publisher also has the option of consulting with two reviewers. Assume that the two reviewers follow the same probabilistic model, and that their reviews are conditionally independent given the actual success or failure. What is the publisher’s optimal strategy; in particular, if he gets one favorable and one unfavorable review, what should his decision be? What is the expected gain from consulting with two reviewers?