Consider the following simple model for disease diagnosis. We make the following assumptions:

- There are $N$ possible symptoms (including test results) which in this problem we will take, unrealistically, to be Boolean – i.e. you either have the symptom or you don’t; the test either succeeds or fails.
- There are $M$ diseases under consideration.
- Symptoms are conditionally independent given the disease.
- Any patient has exactly one diagnosis.
- There are $Q$ different treatments. A patient can be given a single treatment. We will similarly assume, unrealistically, that the effectiveness of a treatment is Boolean; either a treatment entirely cures the disease or it is useless.

The \textit{symptom matrix} is an $M \times N$ matrix $P$ of probabilities: $P[I, J] = \text{Prob}(J|I)$, the probability that a patient exhibits symptom $J$ given that he has disease $I$.

\textbf{Problem 1}

A \textit{patient record} is a Boolean vector of length $N$ indicating the patient's symptoms. Write a function $\text{RecProb}(R, P)$ that takes as arguments a patient record $R$ and a symptom matrix $P$ and returns a vector $D$ of length $M$ such that $D[I] = \text{Prob}(R|I)$ for each disease $I$.

\textbf{Problem 2}

The \textit{frequency} vector is a vector $F$ of length $M$ such that $F[I]$ is the frequency of disease $I$ in the population at large. Write a function $\text{Diagnose}(R, P, F)$ which returns a vector $D$ of length $M$ such that $D[I] = \text{Prob}(I|R)$, the probability that a patient with symptoms $R$ has disease $I$. Use Bayes’ law. Include the normalizing factor.

\textbf{Problem 3}

A \textit{treatment efficacy} matrix is a $Q \times M$ matrix $T$ where $T[I, J]$ is the probability that treatment $I$ will cure disease $J$. Assume that the event that $I$ cures $J$ is independent of the event that $J$ manifests symptom $K$; that is, given that a patient has a particular disease, the effectiveness of the treatment is not affected by the particular symptoms he is manifesting. Write a function $\text{Prognosis}(R, P, F, T)$ which returns a vector $W$ of length $Q$, where $W[I]$ is the probability that a patient with symptoms $R$ will be cured of his disease by treatment $I$. 
Problem 4

A disease cost vector is a vector $C[I]$ of length $M$ indicating the cost of leaving disease $I$ uncured. (We will assume that this depends on the disease, rather than on the symptoms.) A treatment cost vector is a vector $B[I]$ of length $Q$, where indicating the cost of attempting treatment $I$. (Of course, for both of these, “cost” should be interpreted broadly as including all the undesirable consequences.)

Write a function $\text{Benefit}(R,P,F,T,C,B)$ which returns a vector $A[I]$ which is the expected benefit of applying treatment $I$ to a patient with symptoms $R$. Note that the benefit of curing the disease applies only if the disease is cured, whereas the cost of the treatment applies whether or not the disease is cured.