Logic in Computer Science – due October 18, 2004

1. Give a deduction (from \(\emptyset\)) of \(\forall x \ Pxx \rightarrow y = z \rightarrow Pyz.\)

2. Assume that \(x\) does not occur free in \(\alpha\). Show that

\[ \vdash (\alpha \rightarrow \exists x \beta) \leftrightarrow \exists x (\alpha \rightarrow \beta) \]

(i.e. show \(\vdash (\alpha \rightarrow \exists x \beta) \rightarrow \exists x (\alpha \rightarrow \beta)\) and \(\vdash \exists x (\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow \exists x \beta)\)). Note that you do not have to give the deduction, just prove that one exists.

3. A complete calculus has the property that each sentence either has a deduction (from \(\emptyset\)) or a counter-model (i.e., a model in which it is false). For each of the following sentences, either show there is a deduction or give a counter-model.

(a) \(\forall x \ (Qx \rightarrow \forall y Qy)\)
(b) \(\forall z \ (Pz \rightarrow Qz) \rightarrow (\exists x \ Px \rightarrow \forall y Qy)\)
(c) \(\neg \exists y \forall x \ (Px y \leftrightarrow \neg Px x)\)

Assume that \(A \leftrightarrow B\) is an abbreviation for \(\neg ((A \rightarrow B) \rightarrow (B \rightarrow A)).\)