Logic in Computer Science – due Sept. 27, 2004

1. Prove that for all wffs $\alpha$, $\alpha$ is satisfiable iff $\neg\alpha$ is not valid.

2. Is the following formula valid? Justify your answer using a truth table:

\[
(((P \land Q) \rightarrow R) \rightarrow S) \rightarrow (((R \rightarrow Q) \rightarrow P) \rightarrow S).
\]

3. Show that $\{\downarrow\}$ is complete.

4. Say that a set $\Sigma_1$ of wffs is equivalent to a set $\Sigma_2$ of wffs iff for any wff $\alpha$, we have $\Sigma_1 \models \alpha$ iff $\Sigma_2 \models \alpha$. A set $\Sigma$ is independent iff no member of $\Sigma$ is tautologically implied by the remaining members in $\Sigma$. Prove that every finite set of wffs has an independent equivalent subset.

5. Bonus: Find an infinite set that does not have an independent equivalent subset.