Problem 1: (30 points)
Consider the following two sentences in propositional logic:

\[ P \iff (Q \land R). \]
\[ Q \Rightarrow \neg R. \]

A. Convert the above sentences into conjunctive normal form (CNF).

Answer:
1. \( \neg P \lor Q \).
2. \( \neg P \lor R \).
3. \( P \lor \neg Q \lor \neg R \).
4. \( \neg Q \lor \neg R \).

B. Show how the Davis-Putnam procedure finds a valuation satisfying these sentences.

Answer:
Since there are initially no singleton clauses and no pure literals, we try assigning \( P = \text{TRUE} \).
Eliminate clause 3; delete \( \neg P \) from clauses 1 and 2.
New clauses:
1. \( Q \).
2. \( R \).
3. \( \neg Q \lor \neg R \).

Since 1 is a singleton clause, assign \( Q = \text{TRUE} \). Eliminate clause 1, delete \( \neg Q \) from clause 4.
New clauses:
2. \( R \).
4. \( \neg R \).

Since 2 is a singleton clause, assign \( R = \text{TRUE} \). Eliminate clause 2, delete \( \neg R \) from 4. 4 is now the empty clause, so fail.
Return to the beginning. Set \( P = \text{FALSE} \). Eliminate clauses 1 and 2, delete \( P \) from clause 3.
New clauses:
3. \( \neg Q \lor \neg R \).
4. \( \neg Q \lor \neg R \).

\( \neg Q \) is a pure literal. Set \( Q = \text{FALSE} \). Eliminate clauses 3 and 4. \( R \) can be assigned arbitrarily.
Solution: \( P = \text{FALSE}, Q = \text{FALSE}, R \) is arbitrary.
**Problem 2:** (5 points)
(Multiple choice: 1 correct answer)

Suppose that, in the system you built for programming assignment 1, you input to the front end a map coloring problem that has no solution. For example, you specify that there are three countries that all border one another, and that there are only two colors. What happens?

A. The front end will not output any formulas.
B. The Davis-Putnam module will go into an infinite loop.
C. The Davis-Putnam module will output a flag indicating there is no solution.
D. The Davis-Putnam module will output a valuation corresponding to a coloring in which not every country is assigned a color.
E. The Davis-Putnam module will output a valuation corresponding to a coloring in which two neighboring countries have the same color.

**Answer:** C.

**Problem 3:** (10 points)

Consider the following Datalog knowledge base:

A. \( p(X, X) \land p(X, Y) \Rightarrow q(X, Y) \)
B. \( p(a, b) \).
C. \( p(a, c) \).
D. \( p(b, b) \).
E. \( p(b, c) \).

\( X \) and \( Y \) are variables; \( a, b, \) and \( c \) are constants.

What is the result of carrying out forward chaining on this knowledge base?

**Answer:**

Combining A with D and E, under the substitution \( X=b, Y=c \) gives \( q(b, c) \).
Combining A with D and D under the substitution \( X=b, Y=b \) gives \( q(b, b) \).
Problem 4: (30 points)

Let $\mathcal{L}$ be the first-order language over a universe of persons, departments, and courses, containing the following non-logical primitives:

- $\text{teaches}(P, C)$ — Predicate. Person $P$ teaches course $C$.
- $\text{student}(P, C)$ — Predicate. Person $P$ is a student in course $C$.
- $\text{major}(P, D)$ — Predicate. Person $P$ is a major in department $D$.
- $\text{dept}(C, D)$ — Predicate. Course $C$ is in department $D$.
- $\text{cs}$, $\text{english}$, $\text{history}$, $\text{walter}$, $\text{anne}$, $\text{bertrand}$: Constants with the obvious interpretations.

Note: In both this problem and problem 3, you may save yourself writing by using just the initial letter of each symbol. (E.g. use the abbreviation “m(a,h)” instead of writing out “major(anne,history)”.)

Represent the following sentences in $\mathcal{L}$:

A. Walter is taking a CS course.
   **Answer:** $\exists C \text{dept}(C, \text{cs}) \land \text{student(walter,} C\text{)}$.

B. Walter is taking every course that Bertrand is teaching.
   **Answer:** $\forall C \text{teaches(bertrand,} C\text{)} \Rightarrow \text{student(walter,} C\text{)}$.

C. Anne is taking only English and History courses.
   **Answer:** $\forall C \text{student(anne,} C\text{)} \Rightarrow (\text{dept}(C, \text{english}) \lor \text{dept}(C, \text{history}))$.

D. Every major in the CS department is taking some CS course.
   **Answer:** $\forall P \text{major}(P, \text{cs}) \Rightarrow [\exists C \text{dept}(C, \text{cs}) \land \text{student}(P, C)]$.

E. There is a CS course in which all the students are CS majors.
   **Answer:** $\exists C \text{dept}(C, \text{cs}) \land [\forall P \text{student}(P, C) \Rightarrow \text{major}(P, \text{cs})]$.

Problem 5: (25 points)

Consider the following Datalog knowledge base over $\mathcal{L}$: (There are some new constant symbols here.)

1. $\text{student}(P, C) \land \text{major}(P, \text{forestry}) \Rightarrow \text{dept}(C, \text{forestry})$.
   (Forestry students only take courses in the Department of Forestry.)

2. $\text{dept}(C_1, D) \land \text{teaches}(P, C_1) \land \text{teaches}(P, C_2) \Rightarrow \text{dept}(C_2, D)$.
   (If Professor $P$ teaches both $C_1$ and $C_2$, then $C_1$ and $C_2$ are in the same department $D$. In other words, the courses taught by any one professor $P$ are all in the same department(s).)

3. $\text{student}(P, \text{oaks}) \Rightarrow \text{major}(P, \text{forestry})$.
   (All the students in the course "Oaks" are forestry majors.)

4. $\text{student(jack, oaks)}$.

5. $\text{teaches(isabel, oaks)}$.

6. $\text{teaches(isabel, pines)}$.

Show how one can prove “$\text{dept(pines, forestry)}$” using backward chaining.
Goal G0: dept(pines,forestry).
  Match rule (1) under substitution C1=pines.
  Subgoals: G1: student(P1,pines). G2: major(P1,forestry).

Goal G1: student(P1,pines). No match. Fail.

Return to G0.
Match rule (2) under substitution C22=pines, D2=forestry.
  Goal G3: dept(C12,forestry).
  Match rule (1) under substitution C3 = C12.

  Goal G4: student(P3,C12)
  Match fact (4) under binding P3=jack, C12=oaks.
  G4 succeeds.

  Goal G5: major(jack,forestry).
  Match rule (3) under binding P4=jack
  Subgoal G6: student(jack,oaks).

    Goal G6: student(jack,oaks)
    Match fact (4).
    G6 succeeds.
    G5 succeeds.
  G3 succeeds.

Goal G4: teaches(P2,oaks).
Match fact (5) under substitution P2=isabel.
G4 succeeds.

Goal G5: teaches(isabel,pines).
Match fact (6).
G5 succeeds.
G0 succeeds.