Logic in Computer Science – due Sept. 29, 2003

1. Recall our definition of a Ramses number from the first lecture:

The Ramses number $r(x_1, \ldots, x_n)$ is the smallest integer $p$ such that if a complete graph $G$ on $p$ vertices is colored with $n$ colors, then for some $i$, $1 \leq i \leq n$, there must exist a complete subgraph of $G$ with $x_i$ vertices, all of whose edges have the same color.

(a) Write a propositional formula which is satisfiable iff $r(3, 3) > 5$.
(b) If not already in CNF, convert your formula to CNF.
(c) Write a CNF formula which is unsatisfiable iff $r(3, 3) \leq 6$.

2. An $n$-bit comparator is a common circuit in hardware designs. The associated Boolean function is defined as $f(x_1, \ldots, x_n, y_1, \ldots, y_n) = 1$ iff $(x_1 \leftrightarrow y_1) \land (x_2 \leftrightarrow y_2) \land \cdots \land (x_n \leftrightarrow y_n)$. Consider an ROBDD for this function.

(a) Draw the ROBDD for this function when $n=2$ using the variable ordering: $x_1, x_2, y_1, y_2$.
(b) What variable ordering results in the fewest number of nodes? How many nodes (in terms of $n$) does the smallest ROBDD for this function have?
(c) What variable ordering results in the largest number of nodes? How many nodes (in terms of $n$) does the largest ROBDD for this function have?