Logic in Computer Science – due Sept. 22, 2003

1. Prove that for all wffs $\alpha$, $\alpha$ is valid iff $\neg \alpha$ is unsatisfiable.

2. Prove using a truth table that the following formula is valid:

$((P \land Q) \rightarrow R) \rightarrow ((P \rightarrow R) \rightarrow S)$.

3. The dual to disjunctive normal form is conjunctive normal form (CNF). A formula is in CNF if it is a conjunction of formulas, each of which is a disjunction of literals. Prove that for every wff, there exists a tautologically equivalent wff in conjunctive normal form.

4. Show that $\{\}$ is complete.

5. Prove that if $\Sigma$ is finitely satisfiable, then for any wff $\alpha$, at least one of $\Sigma \cup \{\alpha\}$ and $\Sigma \cup \{\neg \alpha\}$ is finitely satisfiable.

6. Let $\Sigma$ be an effectively enumerable set of wffs. Assume that for each wff $\tau$, either $\Sigma \models \tau$ or $\Sigma \models \neg \tau$ (or both). Show that the set of tautological consequences of $\Sigma$ is decidable.