Programming Languages

ML

CSCI-GA.2110-001
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ML overview

- originally developed for use in writing theorem provers
- functional: functions are first-class values
- garbage collection
- strict evaluation (applicative order)
- no coercion
- strong and static typing; powerful type system
  - parametric polymorphism
  - structural equivalence
  - all with type inference!
- advanced module system
- exceptions
- miscellaneous features:
  - datatypes (merge of enumerated literals and variant records)
  - pattern matching
  - `ref` type constructor (like “const pointers” (not “pointers to const”))
Sample SML interactive session

- val k = 5;
  val k = 5 : int  
  user input

- k * k * k;
  val it = 125 : int  
  system response

  ‘it’– last computation

- [1, 2, 3];
  val it = [1,2,3] : int list  
  val it = ["hello","world"] : string list

- 1 :: [2, 3];
  val it = [1,2,3] : int list

- [1, "hello"];
  error
Operations on lists

- null [1, 2];
  val it = false : bool
- null [];
  val it = true : bool
- hd [1, 2, 3];
  val it = 1 : int
- tl [1, 2, 3];
  val it = [2, 3] : int list
- [];
  val it = [] : 'a list  
          this list is polymorphic
Simple functions

A function *declaration*:

```latex
- fun abs x = if x >= 0.0 then x else ~x
val abs = fn : real -> real
```

A function *expression*:

```latex
- fn x => if x >= 0.0 then x else ~x
val it = fn : real -> real
```
Functions, II

- fun length xs = 
  if null xs 
  then 0 
  else 1 + length (tl xs);

val length = fn : 'a list -> int

'a denotes a type variable; length can be applied to lists of any element type

The same function, written in pattern-matching style:

- fun length [] = 0 
  | length (x::xs) = 1 + length xs

val length = fn : 'a list -> int
Type inference and polymorphism

Advantages of type inference and polymorphism:

- frees you from having to write types. A type can be more complex than the expression whose type it is, e.g., `flip`.
- with type inference, you get polymorphism for free:
  - no need to specify that a function is polymorphic
  - no need to "instantiate" a polymorphic function when it is applied
Multiple arguments?

- All functions in ML take exactly one argument
- If a function needs multiple arguments, we can

1. pass a tuple:
   - `(53, "hello"); (* a tuple *)
   val it = (53, "hello") : int * string

   We can also use tuples to return multiple results.

2. use currying (named after Haskell Curry, a logician)
The tuple solution

Another function; takes two lists and returns their concatenation

- fun append1 (xs, ys) = ys
  | append1 (x::xs, ys) = x :: append1 (xs, ys);
val append1 = fn: 'a list * 'a list -> 'a list

- append1 ([1,2,3], [8,9]);
val it = [1,2,3,8,9] : int list
Currying

The same function, written in curried style:

- fun append2 [ ] ys = ys
  | append2 (x::xs) ys = x :: (append2 xs ys);
val append2 = fn: 'a list -> 'a list -> 'a list

Note: $\alpha \rightarrow \beta \rightarrow \delta$ means $\alpha \rightarrow (\beta \rightarrow \delta)$.

- append2 [1,2,3] [8,9];
val it = [1,2,3,8,9] : int list

- val app123 = append2 [1,2,3];
val app123 = fn : int list -> int list

- app123 [8,9];
val it = [1,2,3,8,9] : int list
More partial application

But what if we want to provide the other argument instead, i.e., append \([8,9]\) to its argument?

- here is one way: (the Ada/C/C++/Java way)
  \[
  \text{fun appTo89 xs = append2 xs [8,9]}
  \]

- here is another: (using a higher-order function)
  \[
  \text{val appTo89 = flip append2 [8,9]}
  \]

\text{flip} is a function which takes a curried function \(f\) and returns a function that works like \(f\) but takes its arguments in the reverse order. In other words, it “flips” \(f\)’s two arguments. We define it on the next slide...
fun flip f y x = f x y

The type of flip is \((\alpha \to \beta \to \gamma) \to \beta \to \alpha \to \gamma\). Why?

- Consider \((f \ x)\). \(f\) is a function; its parameter must have the same type as \(x\).

\[
f : A \to B \quad x : A \quad (f \ x) : B
\]

- Now consider \((f \ x \ y)\). Because function application is left-associative, \(f \ x \ y \equiv (f \ x) \ y\). Therefore, \((f \ x)\) must be a function, and its parameter must have the same type as \(y\):

\[
(f \ x) : C \to D \quad y : C \quad (f \ x \ y) : D
\]

- Note that \(B\) must be the same as \(C \to D\). We say that \(B\) must unify with \(C \to D\).

- The return type of flip is whatever the type of \(f \ x \ y\) is. After renaming the types, we have the type given at the top.
Type rules

The type system is defined in terms of inference rules. For example, here is the rule for variables:

\[
(x : \tau) \in E \quad \frac{}{E \vdash x : \tau}
\]

and the one for function calls:

\[
E \vdash e_1 : \tau' \rightarrow \tau \
E \vdash e_2 : \tau' \\
\frac{}{E \vdash e_1 \; e_2 : \tau}
\]

and here is the rule for if expressions:

\[
E \vdash e : \text{bool} \
E \vdash e_1 : \tau \
E \vdash e_2 : \tau \\
\frac{}{E' \vdash \text{if } e \text{ then } e_1 \text{ else } e_2 : \tau}
\]
Passing functions

- fun exists pred [] = false
  | exists pred (x::xs) = pred x orelse
  | exists pred xs;

val exists = fn : ('a -> bool) -> 'a list -> bool

pred is a predicate: a function that returns a boolean

exists checks whether pred returns true for any member of the list

- exists (fn i => i = 1) [2, 3, 4];
  val it = false : bool
Applying functionals

- exists (fn i => i = 1) [2, 3, 4];
val it = false : bool

Now partially apply exists:

- val hasOne = exists (fn i => i = 1);
val hasOne = fn : int list -> bool
- hasOne [3,2,1];
val it = true : bool
fun all pred [ ] = true
    | all pred (x::xs) = pred x andalso all pred xs

fun filter pred [ ] = [ ]
    | filter pred (x::xs) = if pred x
        then x :: filter pred xs
        else filter pred xs

all: \( \alpha \to \text{bool} \to \alpha \text{ list} \to \text{bool} \)

filter: \( \alpha \to \text{bool} \to \alpha \text{ list} \to \alpha \text{ list} \)
let provides local scope:

(* standard Newton-Raphson *)
fun findroot (a, x, acc) =
  let val nextx = (a / x + x) / 2.0
  (* nextx is the next approximation *)
  in
    if abs (x - nextx) < acc * x
    then nextx
    else findroot (a, nextx, acc)
  end
fun quickSort op< [] = []
  | quickSort op< [x] = [x]
  | quickSort op< (a::bs) = 
    let fun partition (left, right, []) = 
        (left, right) (* done partitioning *)
    | partition (left, right, x::xs) = 
        (* put x to left or right *)
        if x < a
        then partition (x::left, right, xs)
        else partition (left, x::right, xs)
    val (left, right) = partition ([] , [a], bs)
    in
    quickSort op< left @ quickSort op< right
    end

quickSort : (α * α → bool) → α list → α list
A variant of Quicksort

fun quickSort op< [] = []
| quickSort op< [x] = [x]
| quickSort op< (a::bs) =
  let fun deposit (x, (left, right)) =
    if x < a
    then (x::left, right)
    else (left, x::right)
  val (left, right) = foldr deposit ([], [a]) bs
  in
  quickSort op< left @ quickSort op< right
end

quickSort : (α * α → bool) → α list → α list
The type system

- **primitive types**: bool, int, char, real, string, unit
- **constructors**: list, array, product (tuple), function, record
- “datatypes”: a way to make new types
- **structural equivalence** (except for datatypes)
  - as opposed to name equivalence in e.g., Ada

- an expression has a corresponding type expression
- the interpreter builds the type expression for each input
- type checking requires that type of functions’ parameters match the type of their arguments, and that the type of the context matches the type of the function’s result
ML records

Records in ML obey structural equivalence (unlike records in many other languages).

A type declaration: *only needed if you want to refer to this type by name*

\[
\text{type vec} = \{ \ x : \text{real}, \ y : \text{real} \ \}\]

A variable declaration:

\[
\text{val v} = \{ \ x = 2.3, \ y = 4.1 \ \}\]

Field selection:

\[
\#x \ v
\]

Pattern matching in a function:

\[
\text{fun dist} \ (x,y) = \\
\qquad \sqrt{\text{pow}(x,2.0) + \text{pow}(y,2.0)}
\]
Tuples

Tuples are actually records.

("I", "Love", "Programming", "Languages")

is actually...

{1="I", 2="Love", 3="Programming", 4="Languages"}

Expression #2 extracts the second element of the tuple. Or, "Love" above.
Datatypes

A **datatype** declaration:

- defines a new type *that is not equivalent to any other type* (name equivalence)
- introduces *data constructors*
  - *data constructors* can be used in patterns
  - they are also values themselves
Datatype example

datatype tree = Leaf of int
  | Node of tree * tree

Tree is a type constructor.

Leaf and Node are data constructors:

- Leaf : int \rightarrow \text{tree}
- Node : \text{tree} \times \text{tree} \rightarrow \text{tree}

We can define functions by pattern matching:

fun sum (Leaf t) = t
  | sum (Node (t1, t2)) = sum t1 + sum t2

Or:

fun sum x = case x of Leaf t => t
  | Node(t1,t2) => sum t1 + sum t2
More on datatypes

Functions accepting data constructors as arguments must provide an exhaustive definition (cover every data constructor for the datatype). Consider again:

```plaintext
datatype tree = Leaf of int
  | Node of tree * tree

fun sum (Leaf t) = t;

stdin:94.5–94.28 Warning: match nonexhaustive
  Leaf t => ...
```
Parameterized datatypes

fun flatten (Leaf t)       = [t]
  | flatten (Node (t1, t2)) =
    flatten t1 @ flatten t2

flatten : tree → int list

datatype 'a gentree =
  Leaf of 'a
  | Node of 'a gentree * 'a gentree

val names = Node (Leaf "this", Leaf "that")

g names : string gentree
The rules of pattern matching

Pattern elements:

- integer literals: 4, 19
- character literals: #'a'
- string literals: "hello"
- data constructors: Node (⋯)
  - depending on type, may have arguments, which would also be patterns
- variables: x, ys
- wildcard: _

Convention is to capitalize data constructors and structure names. Also, start variables and type constructors with lower-case.
More rules of pattern matching

Special forms:

- (){}, – the unit value
- [] – empty list
- [p1, p2, ⋯, pn] means (p1 :: (p2 :: ⋯ (pn :: []))⋯)
- (p1, p2, ⋯, pn) – a tuple
- (p1, _, ⋯, pn) – a partially specified tuple using a wildcard.
- {field1, field2, ⋯ fieldn} – a record
- {field1, field2, ⋯ fieldn, ⋯} – a partially specified record using a wildcard.
- v as p
  – v is a name for the entire pattern p

Example: M as x::xs binds M to the pattern x::xs.
Common idiom: option

**option** is a built-in datatype:

```haskell
datatype 'a option = NONE | SOME of 'a
```

Defining a simple lookup function:

```haskell
fun lookup eq key [] = NONE
    | lookup eq key ((k,v)::kvs) =
      if eq (key, k)
        then SOME v
        else lookup eq key kvs
```

Is the type of **lookup**:

\[(\alpha \times \alpha \rightarrow \text{bool}) \rightarrow \alpha \rightarrow (\alpha \times \beta) \text{list} \rightarrow \beta \text{ option}\]

No! It's slightly more general:

\[(\alpha_1 \times \alpha_2 \rightarrow \text{bool}) \rightarrow \alpha_1 \rightarrow (\alpha_2 \times \beta) \text{list} \rightarrow \beta \text{ option}\]
Another lookup function

We don’t need to pass two arguments when one will do:

```haskell
fun lookup _ [] = NONE
| lookup checkKey ((k,v)::kvs) = 
  if checkKey k
  then SOME v
  else lookup checkKey kvs
```

The type of this lookup:

```
(\alpha \rightarrow \text{bool}) \rightarrow (\alpha \times \beta)\text{list} \rightarrow \beta \text{ option}
```
Useful library functions

- **map**: \((\alpha \rightarrow \beta) \rightarrow \alpha \text{list} \rightarrow \beta \text{list}\)
  
  \[
  \text{map (fn i => i + 1) [7, 15, 3]} \quad \Rightarrow \quad [8, 16, 4]
  \]

- **foldl**: \((\alpha \times \beta \rightarrow \beta) \rightarrow \beta \rightarrow \alpha \text{list} \rightarrow \beta\)
  
  \[
  \text{foldl (fn (a,b) => "(" ^ a ^ " + " ^ b ^ ")") } "0" \quad ["1", "2", "3"] \quad \Rightarrow \quad "(3+(2+(1+0)))"
  \]

- **foldr**: \((\alpha \times \beta \rightarrow \beta) \rightarrow \beta \rightarrow \alpha \text{list} \rightarrow \beta\)
  
  \[
  \text{foldr (fn (a,b) => "(" ^ a ^ " + " ^ b ^ ")") } "0" \quad ["1", "2", "3"] \quad \Rightarrow \quad "(1+(2+(3+0)))"
  \]

- **filter**: \((\alpha \rightarrow \text{bool}) \rightarrow \alpha \text{list} \rightarrow \alpha \text{list}\)
Overloading

Ad hoc overloading interferes with type inference:

```plaintext
define-plus (x, y) = x + y
```

Operator `+` is overloaded, but types cannot be resolved from context (defaults to int).

We can use explicit typing to select interpretation:

```plaintext
define-mix1 (x, y, z) = x * y + z : real
define-mix2 (x: real, y, z) = x * y + z
```
Parametric polymorphism/generics

- a function whose type expression has type variables applies to an infinite set of types
- equality of type expressions means structural not name equivalence
- all applications of a polymorphic function use the same body: no need to instantiate

```ocaml
let val ints = [1, 2, 3];
    val strs = ["this", "that"];;

in

  len ints + (* int list -> int *)
  len strs  (* string list -> int * )

end;
```
An ML *signature* specifies an interface for a module.

```ml
signature STACKS =
sig
  type stack
  exception Underflow
  val empty : stack
  val push : char * stack -> stack
  val pop : stack -> char * stack
  val isEmpty : stack -> bool
end
```
structure Stacks : STACKS =
struct
  type stack = char list
  exception Underflow
  val empty = [ ]
  val push = op:::
  fun pop (c::cs) = (c, cs)
    | pop [] = raise Underflow
  fun isEmpty [] = true
    | isEmpty _ = false
end