Programming Assignment 2

Assigned: Sept. 24
Due: Oct. 8

Note: The assignment is long, but the code is actually very short; perhaps 20 lines in total.

Suppose that $A$ and $B$ are electrically charged objects, located at points $p_A$ and $p_B$ with charges $Q_A$ and $Q_B$. Then the force $\vec{F}_A(B)$ that $B$ exerts on $A$ is the vector

$$\vec{F}_A(B) = \frac{Q_A \cdot Q_B}{|p_A - p_B|^2} \frac{p_A - p_B}{|p_A - p_B|}$$

In the above product, the first factor is the magnitude of the force, which is the product of the charges divided by the distance squared; the second factor is the direction of the force, which is the direction from $B$ to $A$.

If there are several objects $B_1 \ldots B_k$ exerting a force on $A$, then the total force on $A$ is the sum of the forces:

$$\vec{F}_A(\{B_1 \ldots B_k\}) = \sum_{i=1}^{k} \vec{F}_A(B_i)$$

If the charge on $A$ and the position of all the charges is fixed, then the net force is a linear function of vector of charges $\langle \vec{Q} = Q_1 \ldots Q_k \rangle$.

For instance, in two dimensions, we could have the following situation, illustrated in the picture.

| Object | Location | Charge | $|p_A - p_B|$ | Magnitude of $\vec{F}_A(B)$ | $\vec{F}_A(B)$ |
|--------|---------|-------|-------------|----------------|-------------|
| $A$    | $(0,1)$ | 1     | 1           | —              | —           |
| $B_1$  | $(4,4)$ | 50    | 5           | 50/25 = 2      | $2 \cdot (-4,-3)/5 = (-1.60,-1.20)$ |
| $B_2$  | $(1,0)$ | -6    | $\sqrt{2}$ | -6/2 = -3      | $-3 \cdot (-1,1)/\sqrt{2} = (2.12,-2.12)$ |
| $B_3$  | $(-3,1)$ | 36    | 3           | 36/9 = 4       | $4 \cdot (3,0)/3 = (4.00, 0.00)$ |
| Total  |         |       |             | $4.52, -3.32$  | —           |

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Problem 1 (50 points)

Write a function function $F = \text{ForceMatrix}(PA, PB)$ where

- $PA$ is a 2-dimensional column vector of the coordinates of object $A$ of charge 1.
- $PB$ is a $2 \times k$ matrix, where the $i$th column, $PB[:, i]$ is the coordinates of object $B_i$.
- $F$, the value returned is the $2 \times k$ matrix with the property that for any vector of charges $\vec{Q}$, the value $F \cdot \vec{Q}$ is the net force on $A$.

For instance, in the above example, we could call

```matlab
> PA = [0;1];
> PB = [4,1,-3; 4,0,1];
> F = ForceMatrix(PA,PB)
F =
  -0.0320   -0.3536   0.1111
     -0.0240   0.3536   0
> QB = [50; -6; 36];
> F*QB
ans =
     4.5213
    -3.3213
```
Problem 2 (5 points)

Write the following two functions: function \( F = \text{TotalForce}(PA, PB, QB) \) and \( C = \text{PossibleCharge}(PA, PB, TF) \).

In both of these \( PA, PB \) are the same as in problem 1. In \( \text{NetForce} \), the input \( QB \) is a column vector of the charges on \( B \) and the value returned \( F \) is the total force on \( A \), a column vector. In \( \text{PossibleCharge} \), \( TF \) is the total force as a column vector and the value returned \( C \) is a possible charge vector that would give rise to that force. If there are \( k > 2 \) then there are multiple possible answers but your code only has to return one of these. For example, using the same values of \( PA, PB, QB \) we could write,

\[
F = \text{TotalForce}(PA, PB, QB)
\]

\[
F = \begin{bmatrix} 4.5213 \\ -3.3213 \end{bmatrix}
\]

\[
C = \text{PossibleCharge}(PA, PB, F)
\]

\[
C = \begin{bmatrix} 0 \\ -9.3941 \\ 10.8000 \end{bmatrix}
\]

\[
\text{TotalForce}(PA, PB, C)
\]

\[
\text{ans} = \begin{bmatrix} 4.5213 \\ -3.3213 \end{bmatrix}
\]

Having done problem 1, each of these functions should consist of one quite simple line of MATLAB. The code for \( \text{TotalForce} \) should always work, unless \( A \) is at the same position as one of the \( B_i \)'s. The code for \( \text{PossibleCharge} \) may fail in exceptional cases, such as your solution to problem 3.C of problem set 2.

Problem 3 (45 points)

Suppose as before there are \( k \) fixed charges \( B_1 \ldots B_k \) in the plane. You know the locations, but not the value of the charges, and you want to find out the value of the charges. A way to do this is as follows: You take an object \( A \) with charge 1, you put it at various points in the plane, and you measure the net force on it.

Write a function \( \text{function } C = \text{FindCharges}(PA, PB, TF) \) where

- \( PA \) is a \( 2 \times w \) matrix, where the \( i \)th column, \( PA[:,i] \) is the coordinates of the \( i \)th placement of the test charge \( A \). The dimension \( w \) is the number of different placements you try.
- \( PB \) is the locations of the charges \( B_1 \ldots B_k \), as above.
- \( F \) is a \( 2 \times q \) matrix, where the \( i \)th column \( F[:,i] \) is the total force on \( A \) in its \( i \)th placement.
- The value returned \( C \) is the \( k \)-dimensional column vector of charges on the \( B_i \).

Hint: Look up the Matlab \texttt{reshape} function.
For instance, in the above example, we could call

```matlab
> PA = [0,2;1,0];
> PB = [4,1,-3; 4,0,1];
> TF(:,1) = TotalForce(PA(:,1),PB,QB);
> TF(:,2) = TotalForce(PA(:,2),PB,QB);
> C = FindCharges(PA,PB,TF)
C =
    50.0000
   -6.0000
    36.0000
```