V22.0453-001 Honors Theory of Computation

Problem Set 3

All problems are worth 10 points. Due on Tue Nov 9, 2010.

Problem 1

A *Turing machine with stay put instead of left* is similar to an ordinary Turing machine, but the transition function is of the form:

$$Q \times \Gamma \mapsto Q \times \Gamma \times \{R, S\}$$

At each point the machine can move its head to right or let it stay in the same position. Show that this Turing machine variant is *not* equivalent to the usual version. What class of languages do these machines recognize ?

Problem 2

Show that the collection of Turing-recognizable languages is closed under the operation of (a) union (b) concatenation (c) star and (d) intersection. What about complementation operation ?

Problem 3

Show that the following language is decidable:

$$INFINITE_{CFG} = \{\langle G\rangle \mid G \text{ is a context-free grammar such that } L(G) \text{ is infinite}\}$$

Problem 4

Show that the following language is decidable:

$$L = \{\langle R, S\rangle \mid R \text{ and } S \text{ are regular expressions such that } L(R) \subseteq L(S)\}$$

Problem 5

Show that the following language is undecidable:

$$A = \{\langle M\rangle \mid M \text{ is a Turing machine that accepts } w \text{ whenever it accepts } w^R\}$$

Here w^R denotes the reverse of string w.

Problem 6 In this problem, we explore the notion of *oracle reducibility.* If A is a language, then a *Turing machine with oracle* A is a Turing machine with a "magical" subroutine that decides membership in A. In other words, the subroutine, when given a string w, tells the machine whether or not $w \in A$. Let

$$HALT_{TM} = \{\langle M, x\rangle \mid M \text{ is a Turing machine that halts on } x\}$$

Show that there is a Turing machine with oracle $HALT_{TM}$ that decides the following problem with only *two* questions to the oracle: Given three (machine, input) pairs $\langle M_1, x_1\rangle, \langle M_2, x_2\rangle, \langle M_3, x_3\rangle$, decide for each pair whether the Turing machine halts on the corresponding input.

Note: This is trivial if one allows *three* questions. Just ask the oracle whether $\langle M_i, x_i\rangle \in HALT_{TM}$ for $i = 1, 2, 3$.