CSCI-UA.0453-001: Theory of Computation

Problem Set 4

All problems are worth 10 points. Collaboration is allowed, but you must write your own solutions. Write the names of your collaborators (and your own!).
Unless stated otherwise, you must show all intermediate steps and give proper justification or proof.

Problem 1

Answer whether each of the following languages is decidable, and justify your answer. You may find Rice's Theorem useful for some parts.

1. $\{\langle M, w, t\rangle : M \text{ halts on } w \text{ in } t \text{ steps}\}$
2. $\{\langle M\rangle : \varepsilon \in L(M)\}$
3. $\{\langle M\rangle : M \text{ halts on } \varepsilon\}$
4. $\{\langle M\rangle : M \text{ halts on some input}\}$
5. $\{\langle M\rangle : L(M) \text{ is context-free}\}$

Problem 2

For each of the following statements, state whether it is TRUE or FALSE, and justify your answer.

1. $\exists$ constants $c < d$ such that $n^d = O(n^c)$
2. $10^{10} \cdot n^{1000} = O(2^{0.001n})$
3. $n^{10} = O(2^{\log^2 n})$
4. $2^{\sqrt{\log n}} = O(\sqrt{n})$
5. $n^{\log n} = O(2^{\sqrt{n}})$

Problem 3

Show that P is closed under the star operation (**Hint**: Use dynamic programming.) Recall that for a language L,

$$L^* = \{x_1 x_2 \dots x_k \mid k \geq 0,\ x_i \in L\ \forall 1 \leq i \leq k\}$$

Problem 4

Let DOUBLE-SAT $= \{\langle \phi\rangle \mid \phi$ is a boolean formula that has at least two satisfying assignments$\}$. Show that DOUBLE-SAT is NP-complete.

Problem 5

Problem 7.26 on Page 324 of Sipser (this is about the $\neq$-SAT problem).

Problem 6

Problem 7.27 on Page 325 of Sipser (this is about the MAX-CUT problem).