Honors Analysis of Algorithms

Problem Set 6

Collaboration is allowed, but you must write your own solutions.

Problem 1

Show that P is closed under the star operation (*Hint: Use dynamic programming.*) Recall that for a language L,

$$L^* = \{x_1 x_2 \dots x_k \mid k \geq 0,\ x_i \in L\ \forall 1 \leq i \leq k\}$$

Problem 2

Show that these problems are NP-complete (you can assume that SAT and CLIQUE are NP-complete).

1. DOUBLE-SAT $= \{\langle\phi\rangle \mid \phi$ is a boolean formula that has at least two satisfying assignments$\}$.
2. HALF-CLIQUE $= \{\langle G\rangle \mid G$ is a graph with a clique of size at least $|G|/2\}$.

Problem 3

Let $\{x_1, x_2, \dots, x_n\}$ be boolean variables. A literal is either a variable or its negation, i.e. x_i or $\overline{x}_i$. A clause is logical OR of one or more distinct literals. The size of a clause is the number of literals in it. A 2CNF formula ϕ is a collection of m clauses (possibly with repetition),

$$\phi = (C_1, C_2, \dots, C_m)$$

where each C_i is of size at most two. Let MAX-2SAT be the following decision problem: Given a pair (ϕ, k) where ϕ is a 2CNF formula with n variables and m clauses, and k is a positive integer such that $k \leq m$, decide whether there exists an assignment to the n boolean variables that satisfies at least k clauses. Show that MAX-2SAT is NP-complete, by giving a polynomial time reduction from VERTEX COVER.

Recall that VERTEX COVER is a problem where, given an undirected graph $G = (V, E)$, with $|V| = n$, and given a positive integer $\ell \leq n$, one needs to decide whether G has a vertex cover of size at most ℓ. A vertex cover is a subset $V' \subseteq V$ such that for every edge in E, at least one of its endpoints is included in V'.

Hint: To every vertex in the graph, assign a boolean variable which is intended to be TRUE if and only if the vertex is included in the vertex cover. Add clauses of size two corresponding to the edges, and clauses of size one corresponding to the vertices. The clauses corresponding to edges may need to be repeated a number of times.

Problem 4

Show that these problems are NP-complete:

1. Problem 5 in http://cs.nyu.edu/web/Academic/Graduate/exams/sample/algs_f09.pdf
2. Problem 6 in http://cs.nyu.edu/web/Academic/Graduate/exams/sample/algs_f08.pdf

Problem 5

Solve Problem 2 in http://cs.nyu.edu/web/Academic/Graduate/exams/sample/algs_f08.pdf