Honors Analysis of Algorithms

Problem Set 5

Collaboration is allowed, but you must write your own solutions. Proofs of correctness are a must.

Problem 1 (Randomized Algorithms)

Solve: [Kleinberg Tardos] Chapter 13, problem 1, page 782.

Problem 2 (Randomized Algorithms)

Solve: [Kleinberg Tardos] Chapter 13, problem 10, page 789.

Hint: Consider an agent such that there are k agents with a higher bid than her. What is the probability that her bid results in an update of b^*?

Problem 3 (Randomized Algorithms)

Solve: [Kleinberg Tardos] Chapter 13, problem 11, page 789.

Problem 4

Solve Problem 4 from: http://www.cs.nyu.edu/courses/fall07/G22.3520-001/ps2.pdf

Note: This is the problem about the algorithm $A(S)$. You can assume that R is chosen such that $R \neq \emptyset$ and $R \neq S$ (so that the recursive calls terminate in at most $|S|$ steps). What is an upper bound on a value output by the algorithm? You can first solve the problem assuming that when a random subset $R \subseteq S$ is chosen, it is always the case that $\frac{|S|}{3} \leq |R| \leq \frac{2|S|}{3}$. Now you can relax the assumption noting that the probability that $|R|$ is outside of this range is at most $2^{-\alpha|S|}$ for some universal constant α.

Problem 5

In this problem, we explore the notion of *oracle reducibility.* If A is a language, then a *Turing machine with oracle* A is a Turing machine with a "magical" subroutine that decides membership in A. In other words, the subroutine, when given a string w, tells the machine whether or not $w \in A$. Let

$$\mathrm{HALT}_{\mathrm{TM}} = \{\langle M, x\rangle \mid M \text{ is a Turing machine that halts on } x\}.$$

Show that there is a Turing machine with oracle $\mathrm{HALT}_{\mathrm{TM}}$ that decides the following problem with only *two* questions to the oracle: Given three (machine, input) pairs $\langle M_1, x_1\rangle, \langle M_2, x_2\rangle, \langle M_3, x_3\rangle$, decide for each pair whether the Turing machine halts on the corresponding input.

Note: This is trivial if one allows *three* questions. Just ask the oracle whether $\langle M_i, x_i \rangle \in \mathrm{HALT}_{\mathrm{TM}}$ for $i = 1, 2, 3$.

Problem 6

Show that the collection of Turing-recognizable languages is closed under the operation of (a) union (b) concatenation (c) star and (d) intersection. What about complementation operation ?