Lecture 2:
Color, Filtering & Edges

Prof. Rob Fergus

Slides: S. Lazebnik, S. Seitz, W. Freeman, F. Durand, D. Forsyth, D. Lowe, B. Wandell, S.Palmer, K. Grauman

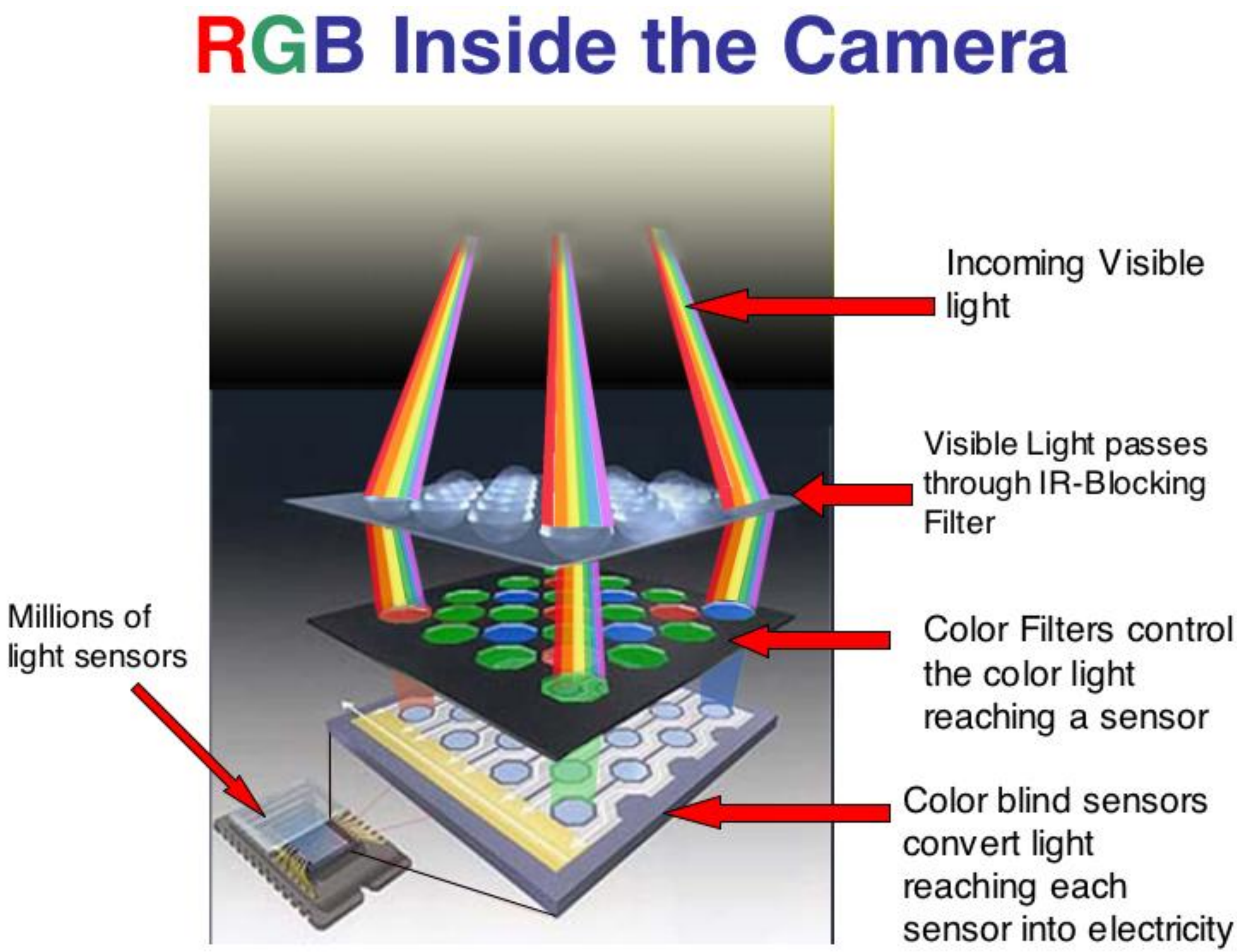
Color

What is color?

- Color is a psychological property of our visual experiences when we look at objects and lights,
  not a physical property of those objects or lights
  (S. Palmer, *Vision Science: Photons to Phenomenology*)

- Color is the result of interaction between physical light in the environment and our visual system

Color Camera Sensor

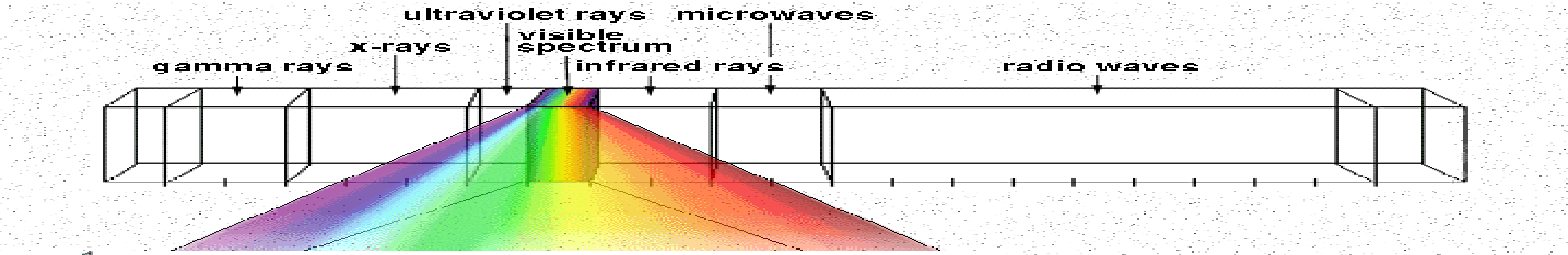


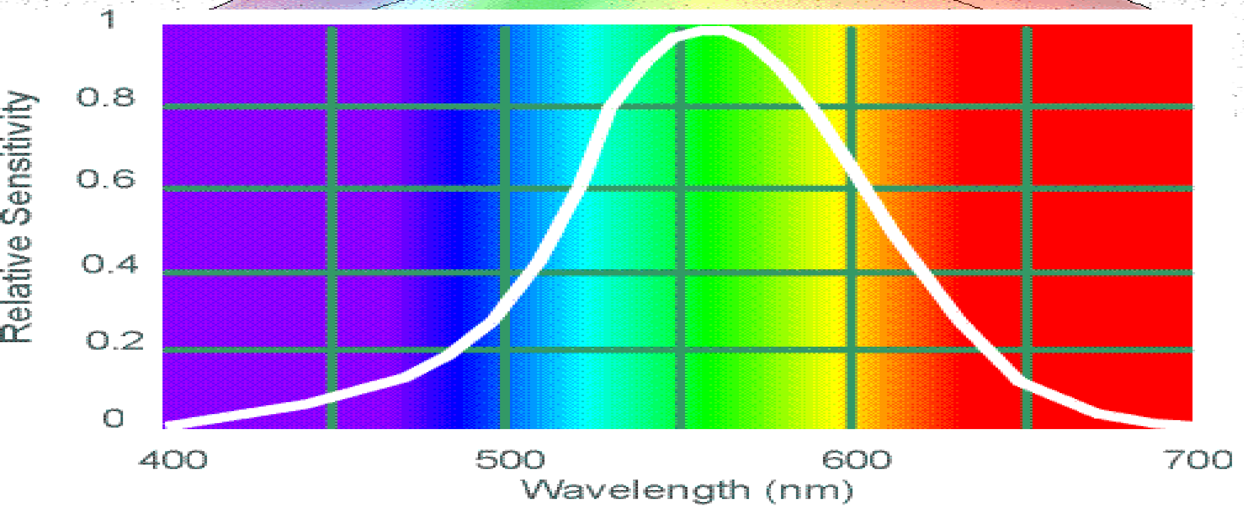
http://www.photoaxe.com/wp-content/uploads/2007/04/camera-sensor.jpg

Overview of Color

- Physics of color
- Human encoding of color
- Color spaces
- White balancing

Electromagnetic Spectrum



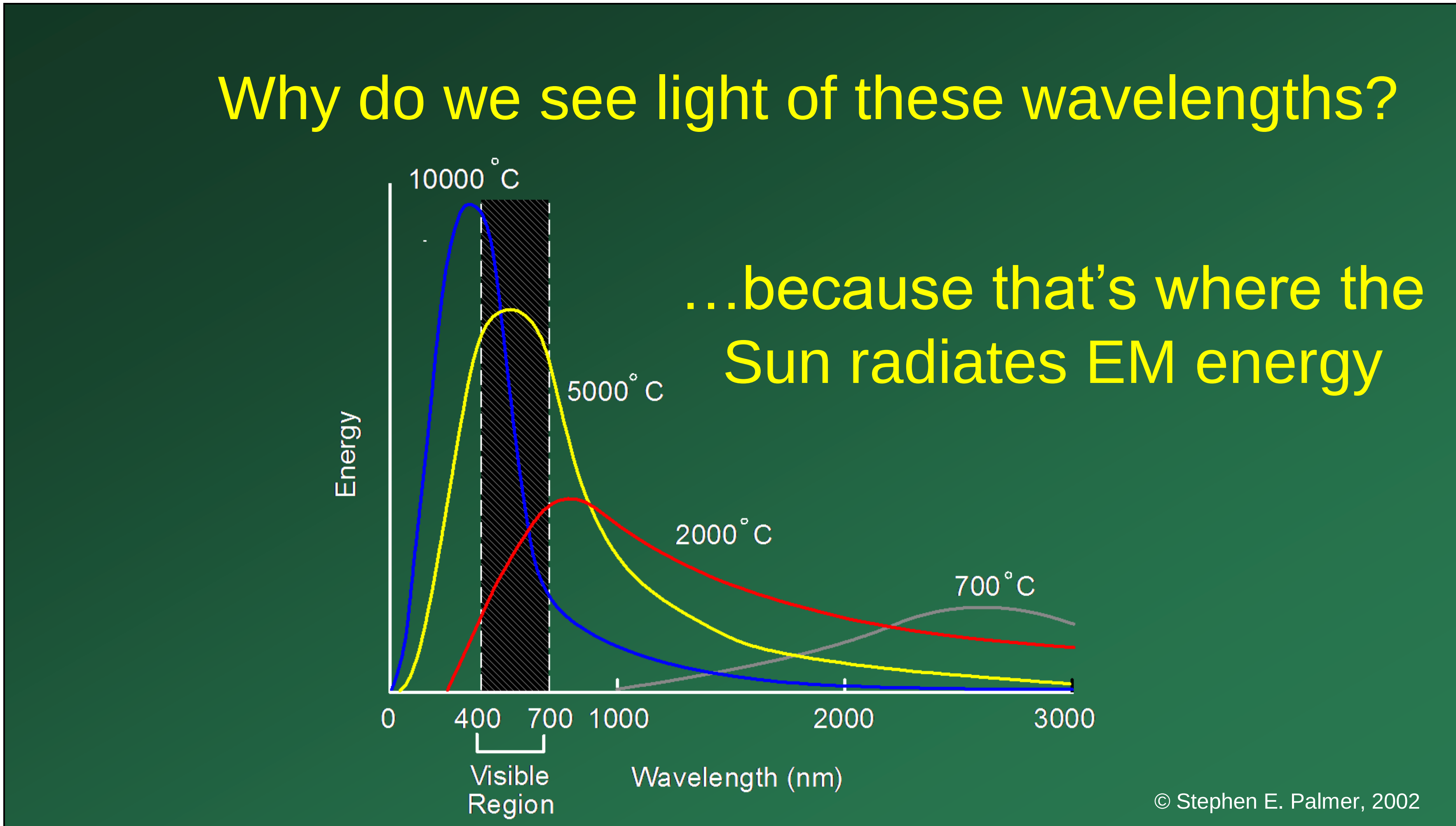


Human Luminance Sensitivity Function

http://www.yorku.ca/eye/photopik.htm

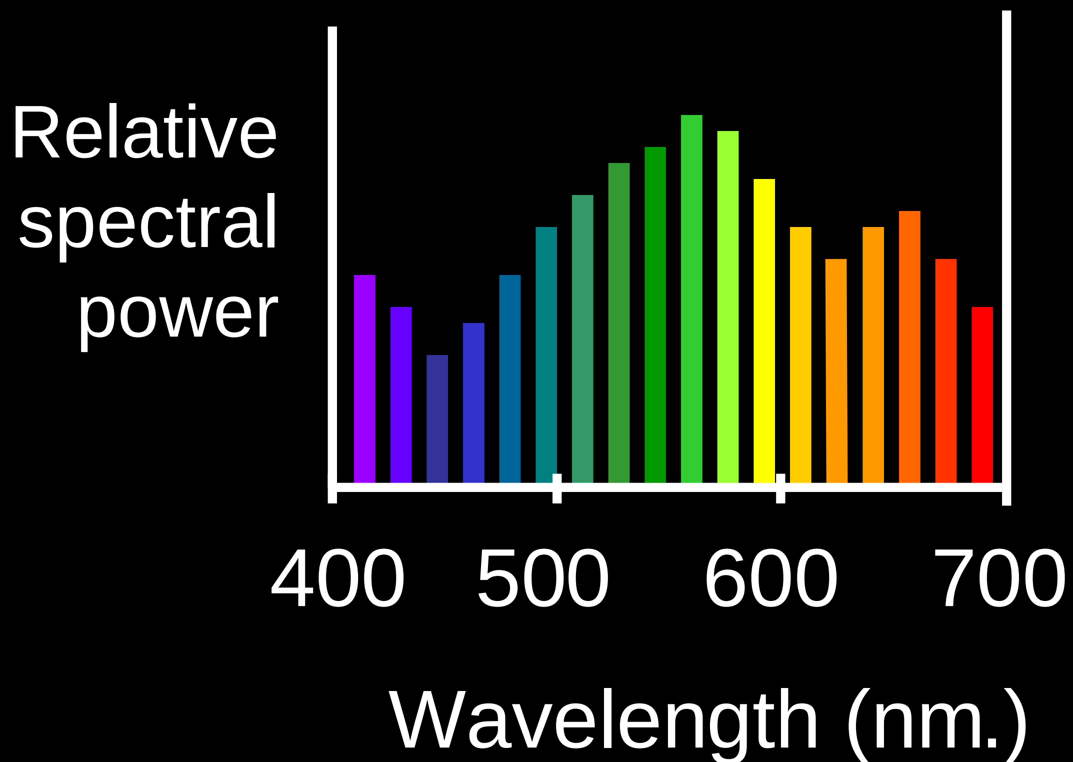
Visible Light

Plank's law for Blackbody radiation
Surface of the sun: ~5800K



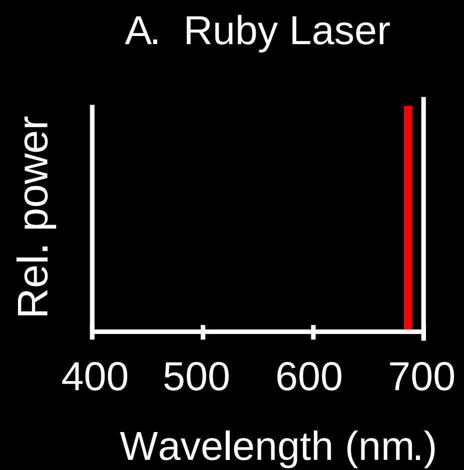
The Physics of Light

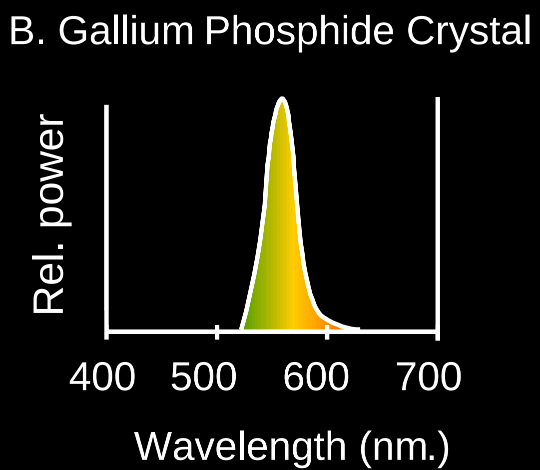
Any source of light can be completely described physically by its spectrum: the amount of energy emitted (per time unit) at each wavelength 400 - 700 nm.

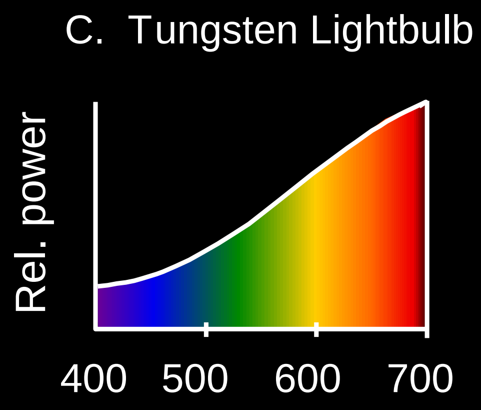


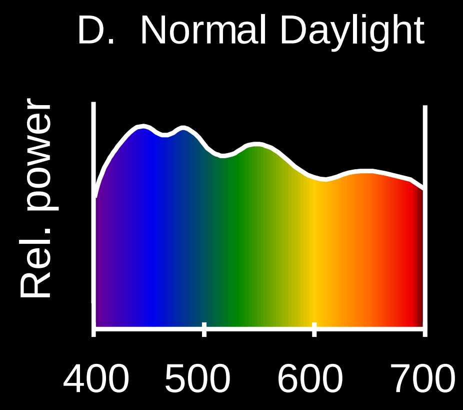
The Physics of Light

Some examples of the spectra of light sources



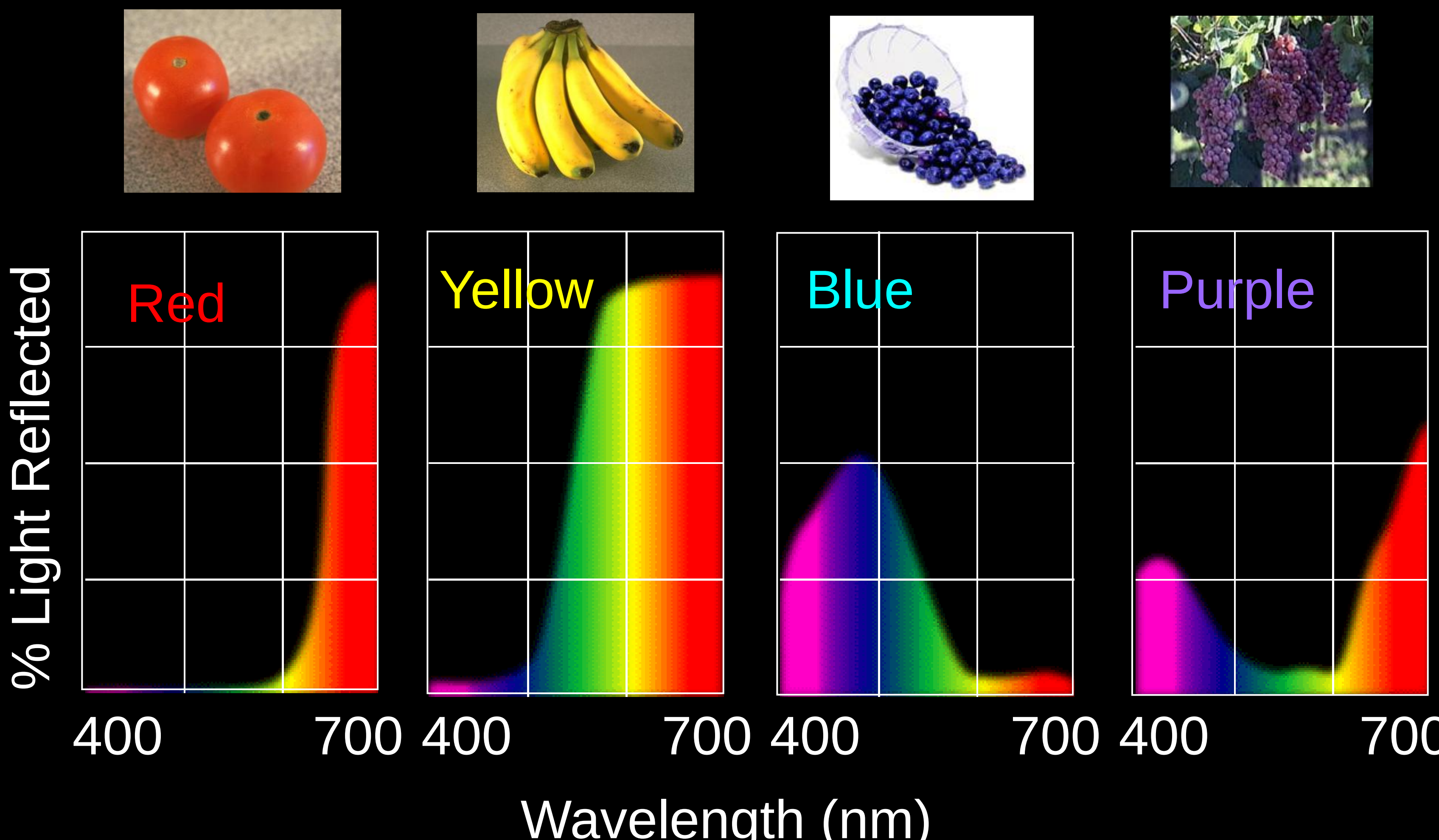






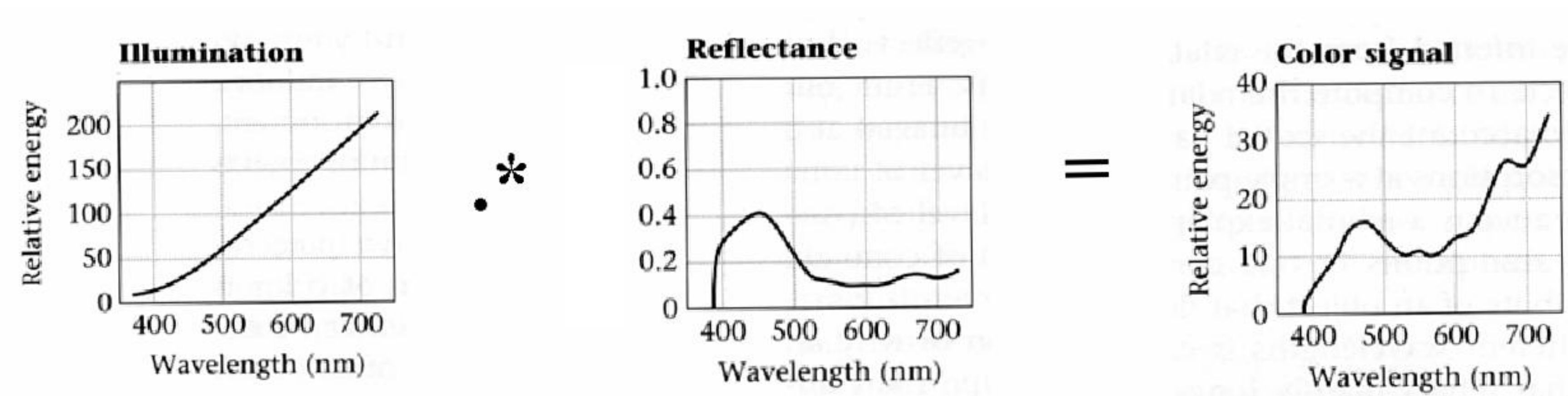
The Physics of Light

Some examples of the reflectance spectra of surfaces



Interaction of light and surfaces

- Reflected color is the result of interaction of light source spectrum with surface reflectance
- Spectral radiometry
  - All definitions and units are now "per unit wavelength"
  - All terms are now "spectral"



From Foundation of Vision by Brian Wandell, Sinauer Associates, 1995

Interaction of light and surfaces

- What is the observed color of any surface under monochromatic light?

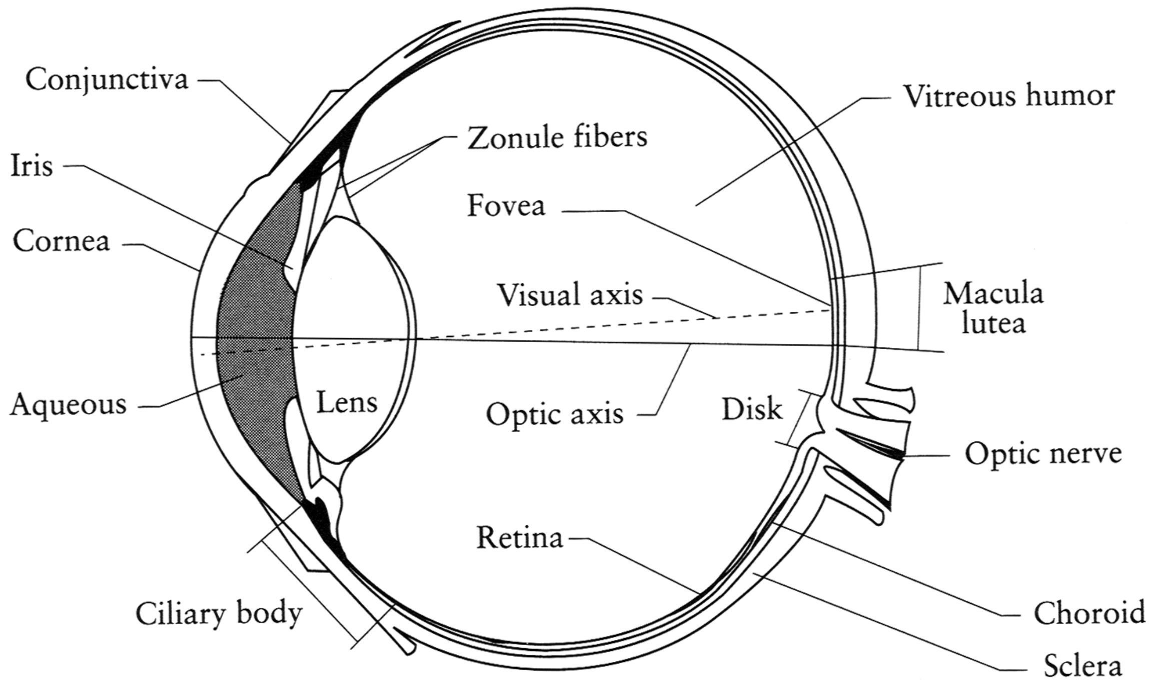
[Olafur Eliasson, *Room for one color*]

Slide by S. Lazebnik

Overview of Color

- Physics of color
- Human encoding of color
- Color spaces
- White balancing

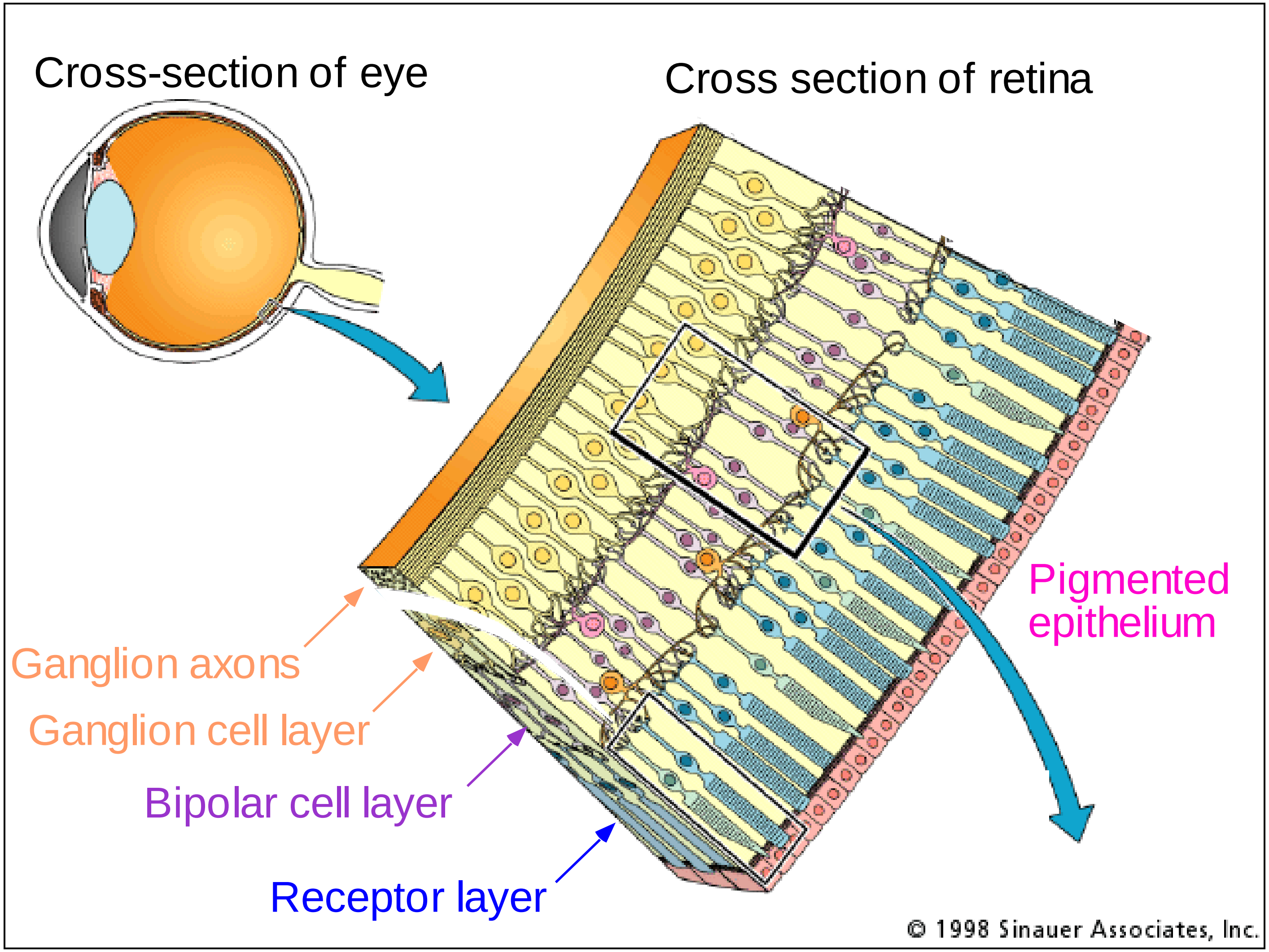
The Eye



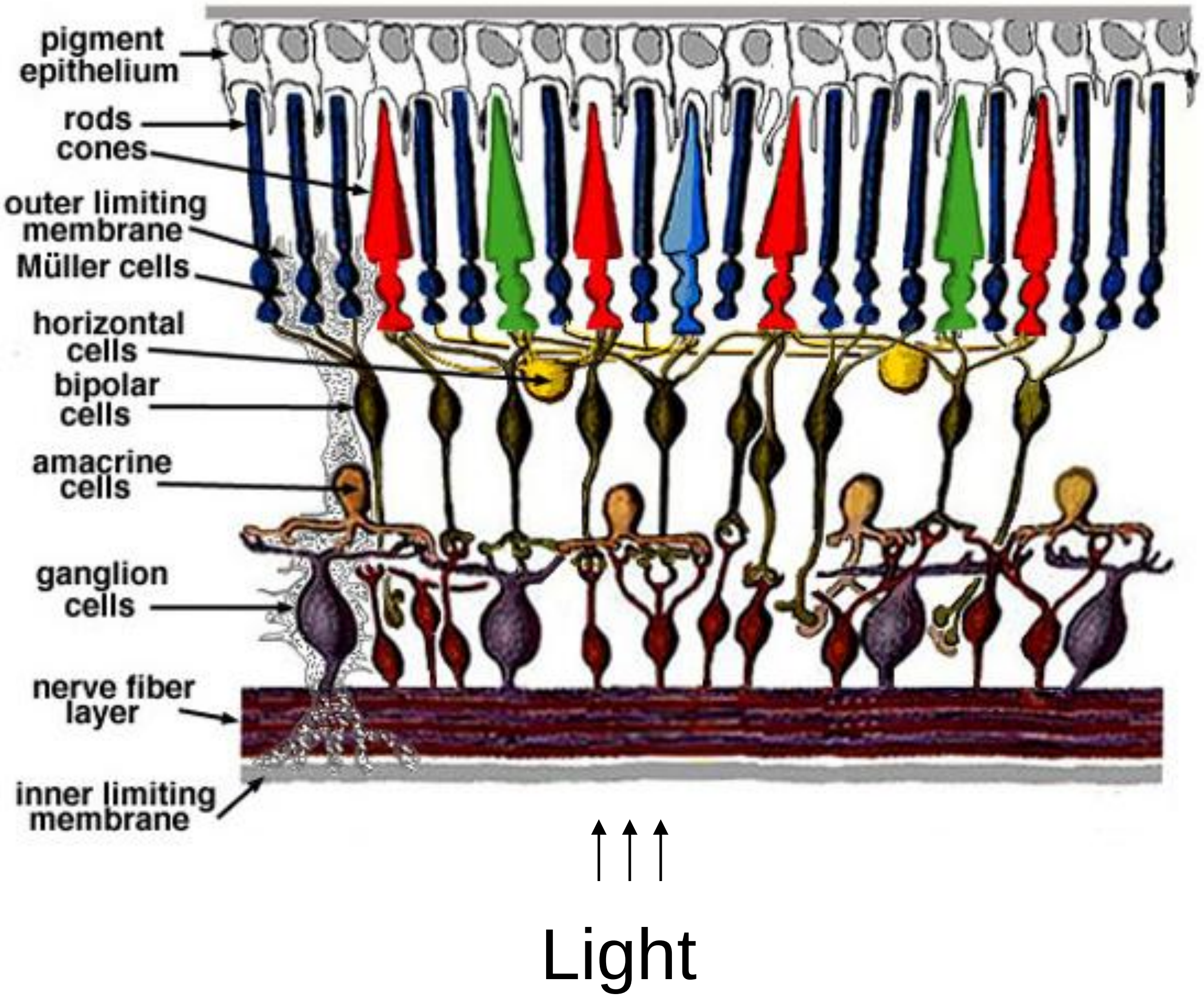
The human eye is a camera!

- **Iris** - colored annulus with radial muscles
- **Pupil** - the hole (aperture) whose size is controlled by the iris
- What’s the “film”?
  - photoreceptor cells (rods and cones) in the **retina**

The Retina


Cross-section of eye
Cross section of retina
Ganglion axons
Ganglion cell layer
Bipolar cell layer
Receptor layer
Pigmented epithelium
© 1998 Sinauer Associates, Inc.

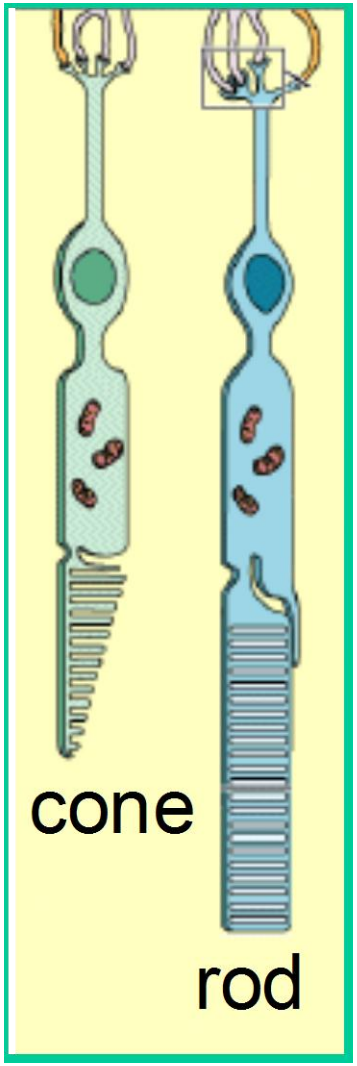
Retina up-close



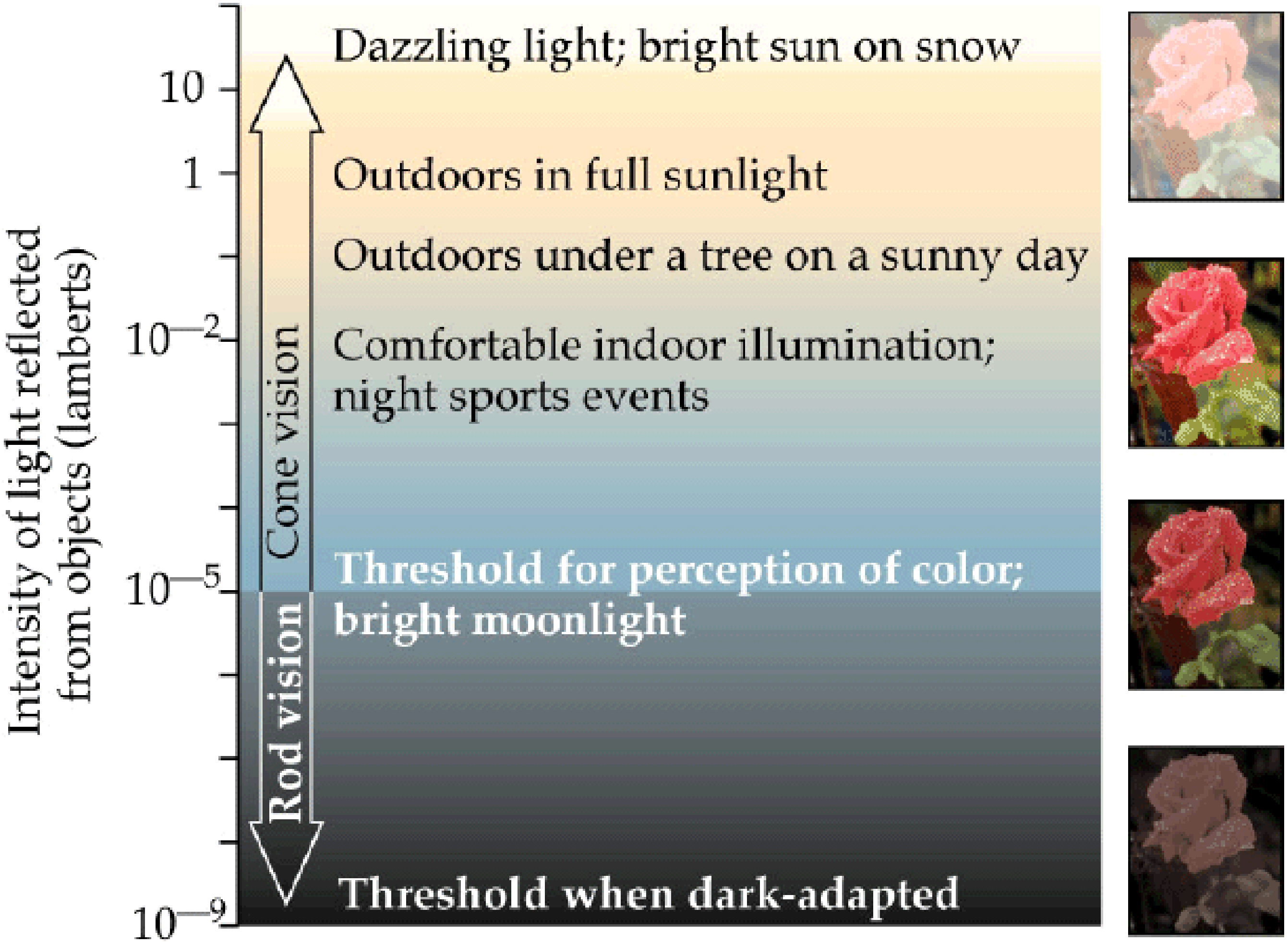
Two types of light-sensitive receptors

Cones
cone-shaped
less sensitive
operate in high light
color vision

Rods
rod-shaped
highly sensitive
operate at night
gray-scale vision



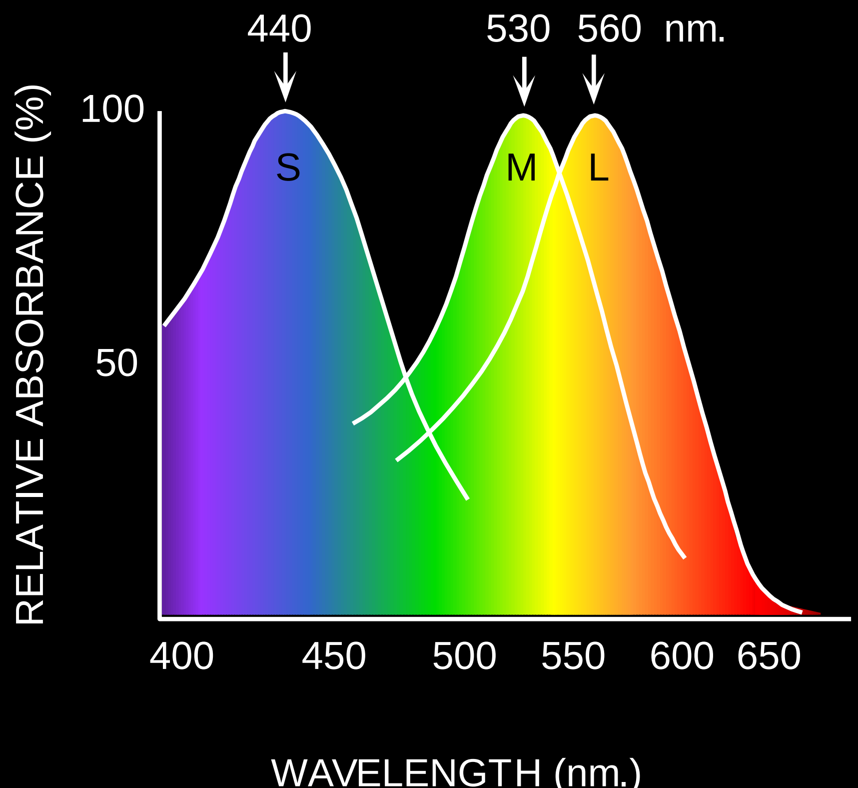
Rod / Cone sensitivity

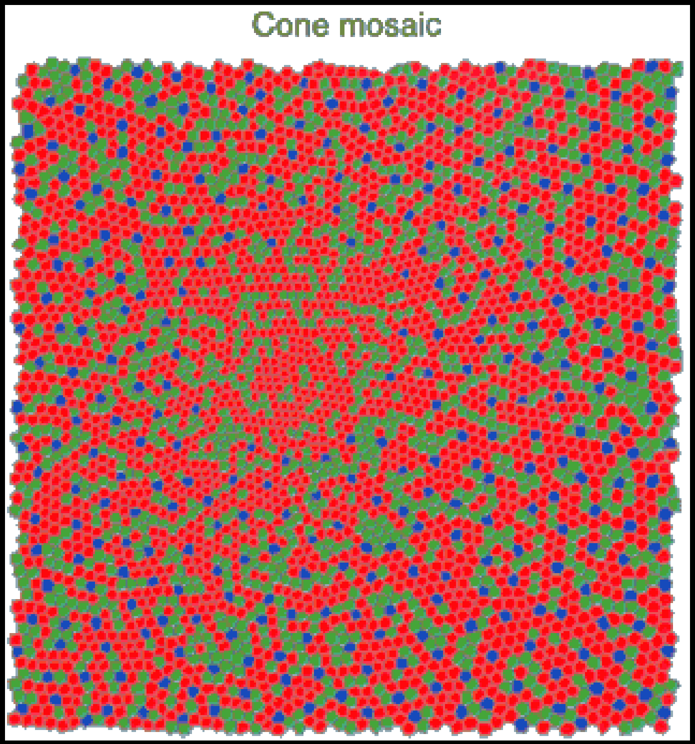


The famous sock-matching problem…

Physiology of Color Vision

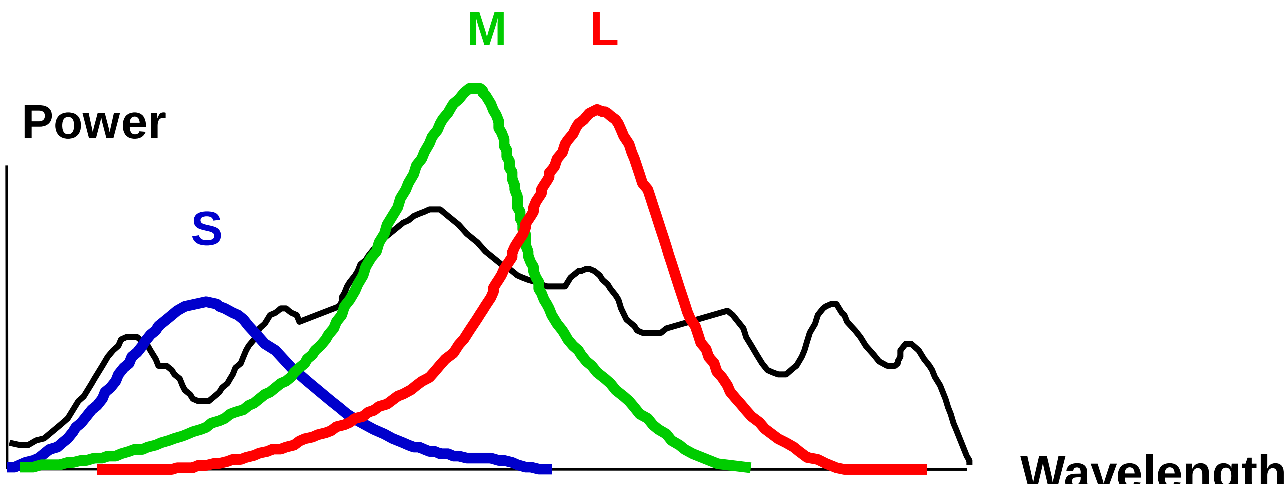
Three kinds of cones:





- Why are M and L cones so close?
- Are are there 3?

Color perception



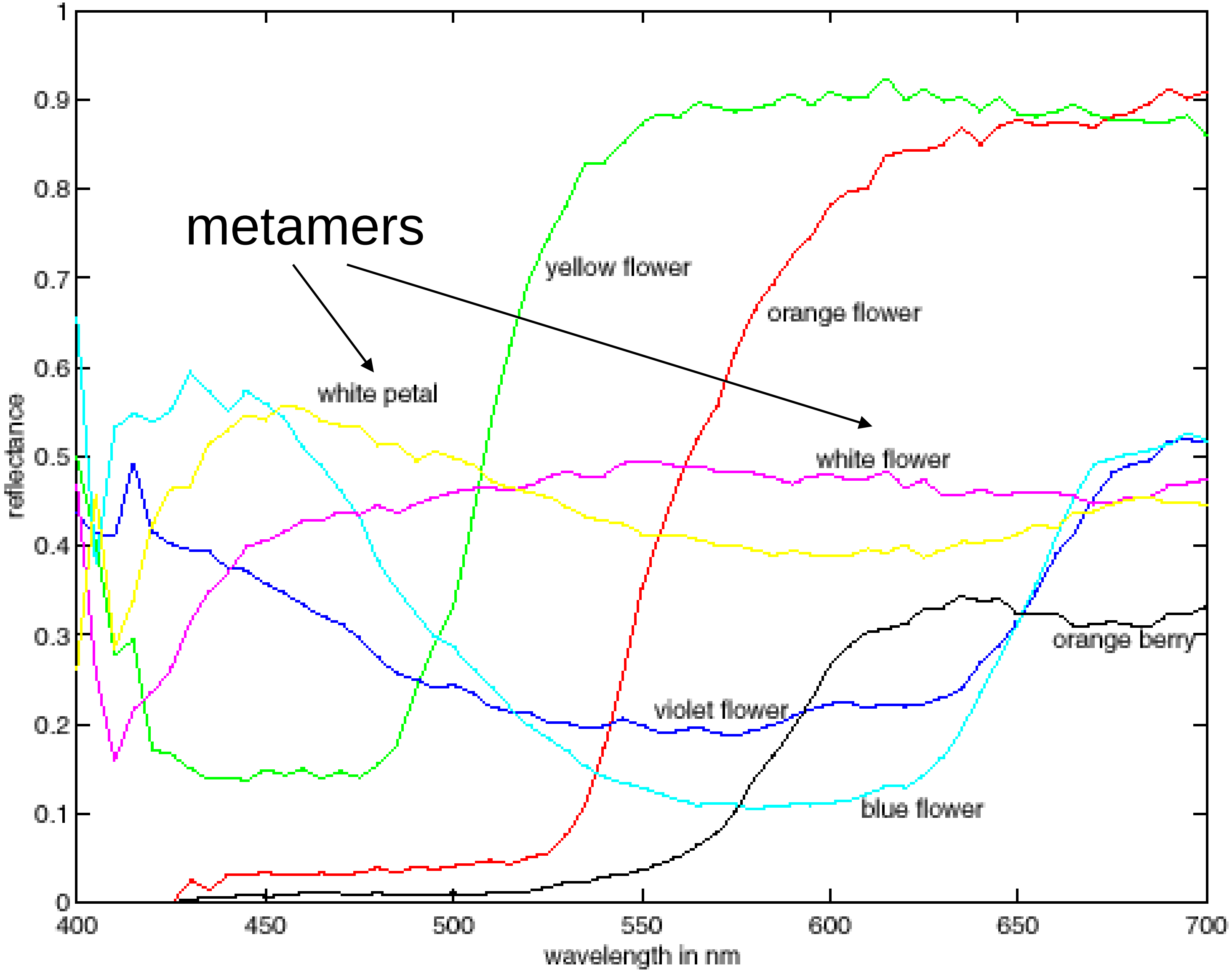
Rods and cones act as filters on the spectrum

- To get the output of a filter, multiply its response curve by the spectrum, integrate over all wavelengths
  - Each cone yields one number

- Q: How can we represent an entire spectrum with 3 numbers?
- A: We can't! Most of the information is lost.
  - As a result, two different spectra may appear indistinguishable
    - » such spectra are known as **metamers**

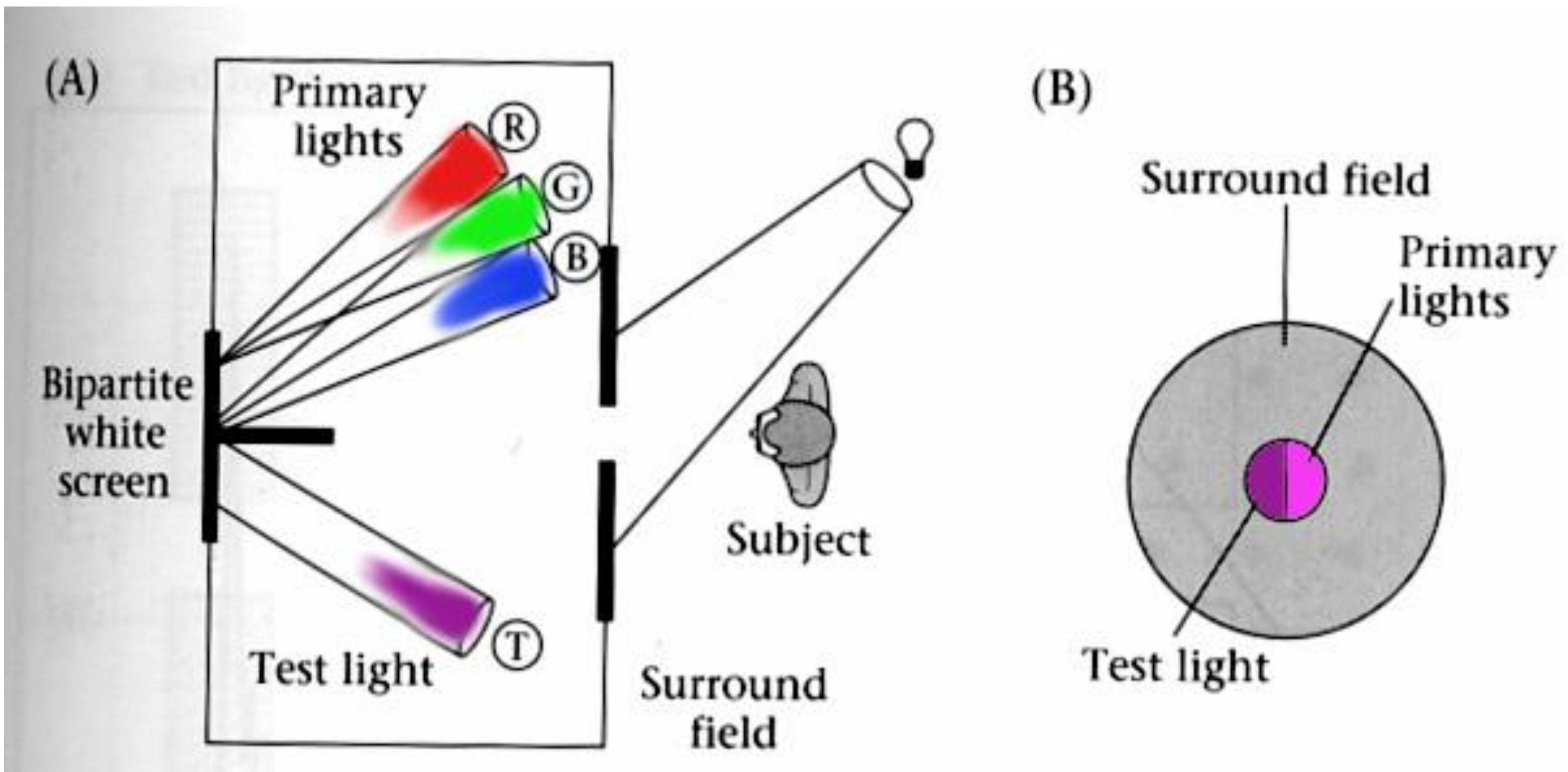
Slide by Steve Seitz

Spectra of some real-world surfaces



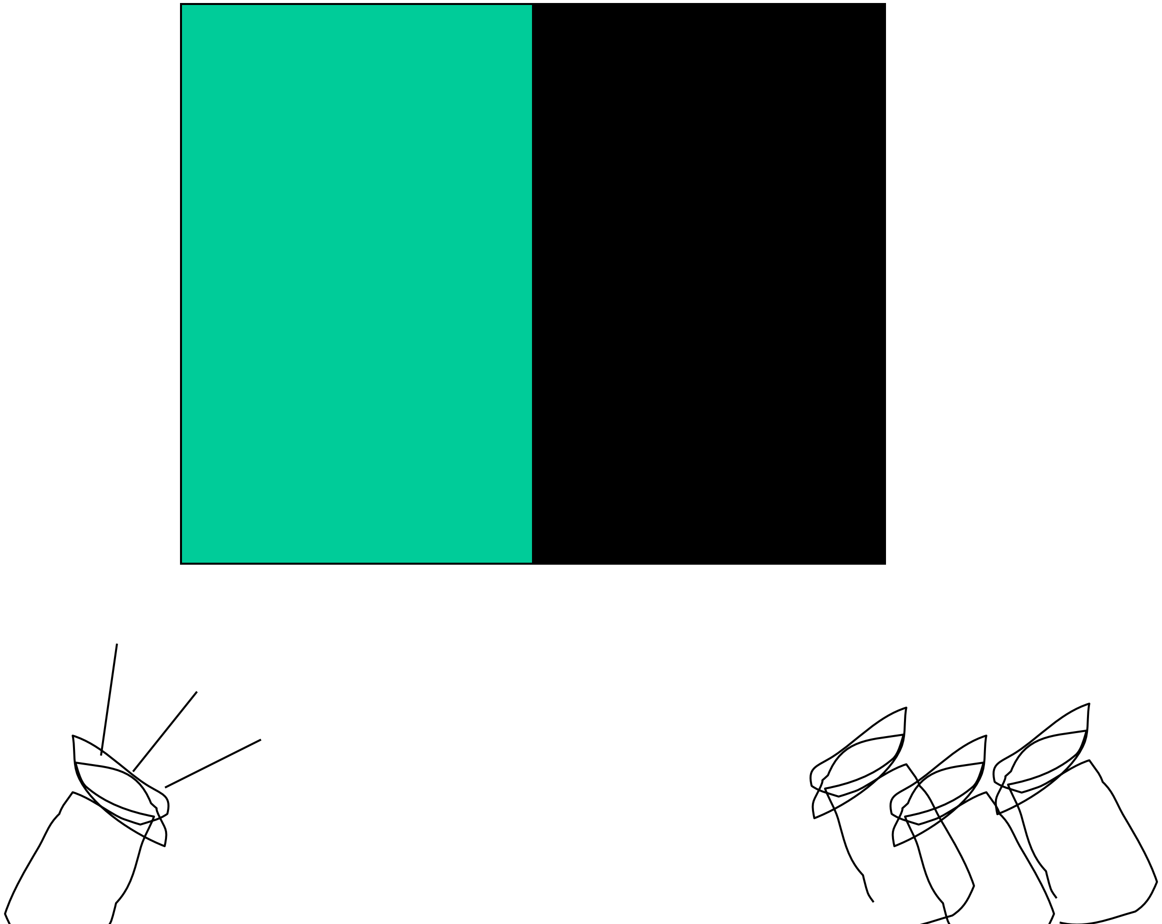
Standardizing color experience

- We would like to understand which spectra produce the same color sensation in people under similar viewing conditions
- Color matching experiments



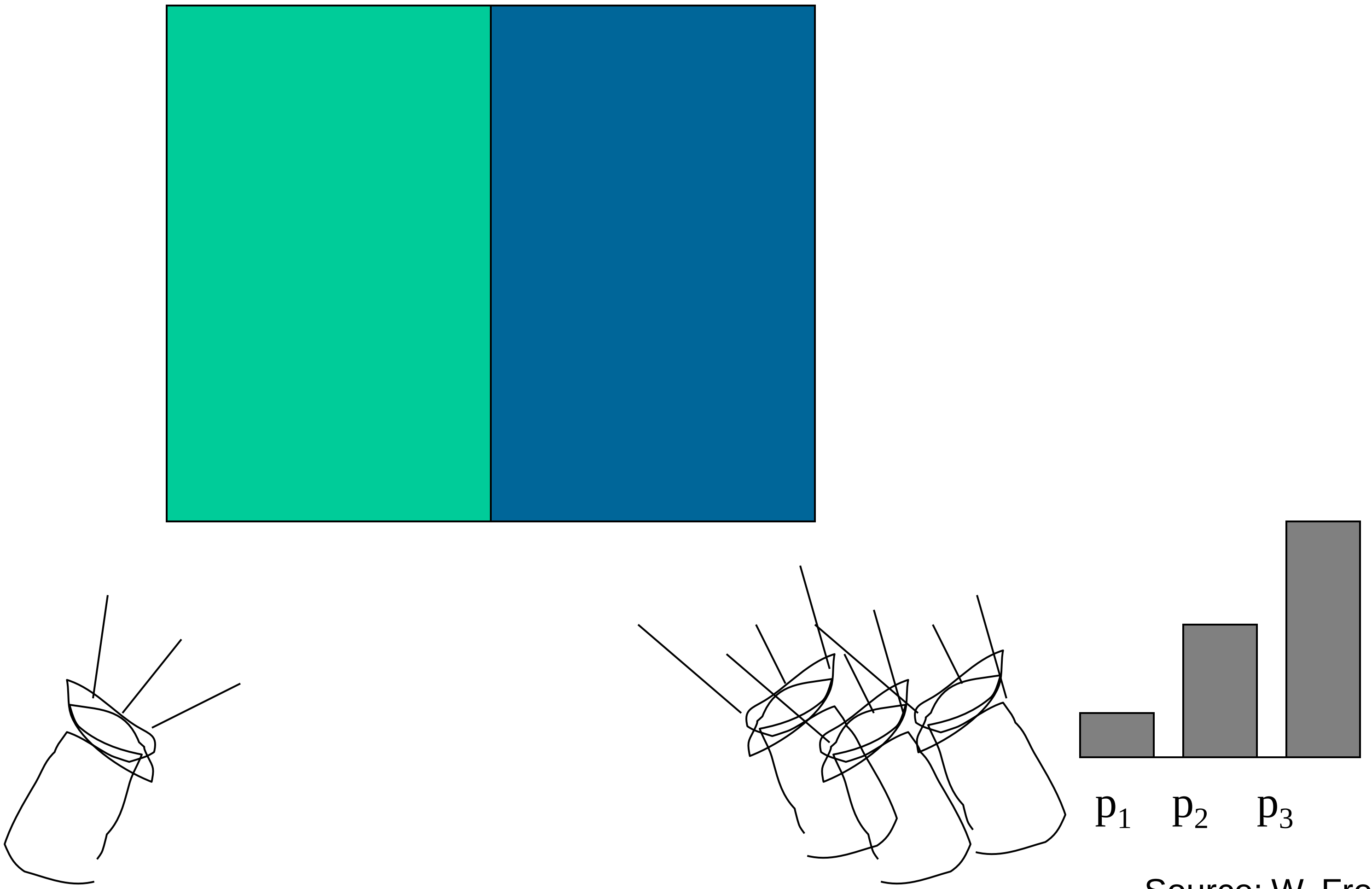
Foundations of Vision, by Brian Wandell, Sinauer Assoc., 1995

Color matching experiment 1



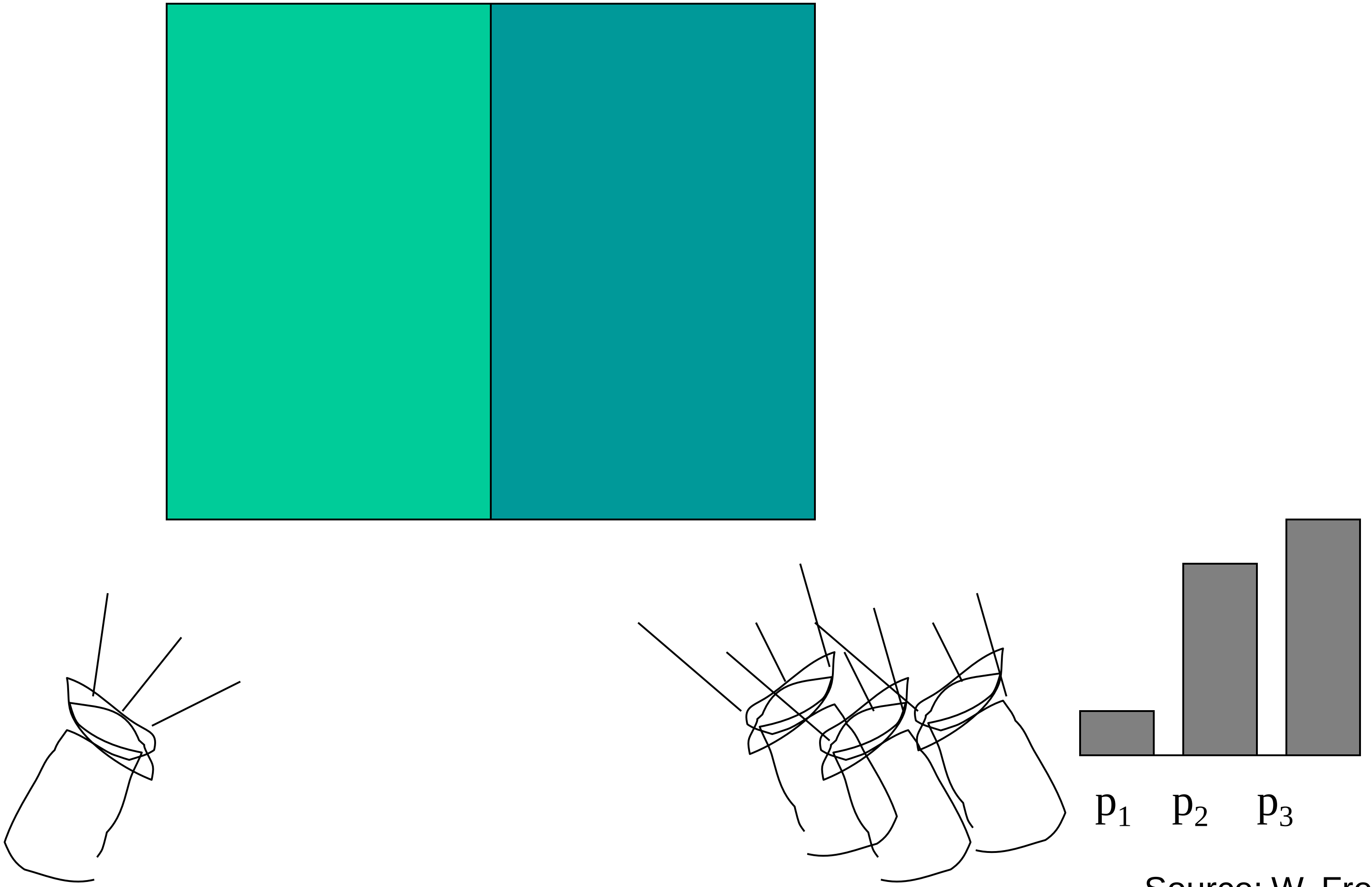
Source: W. Freeman

Color matching experiment 1



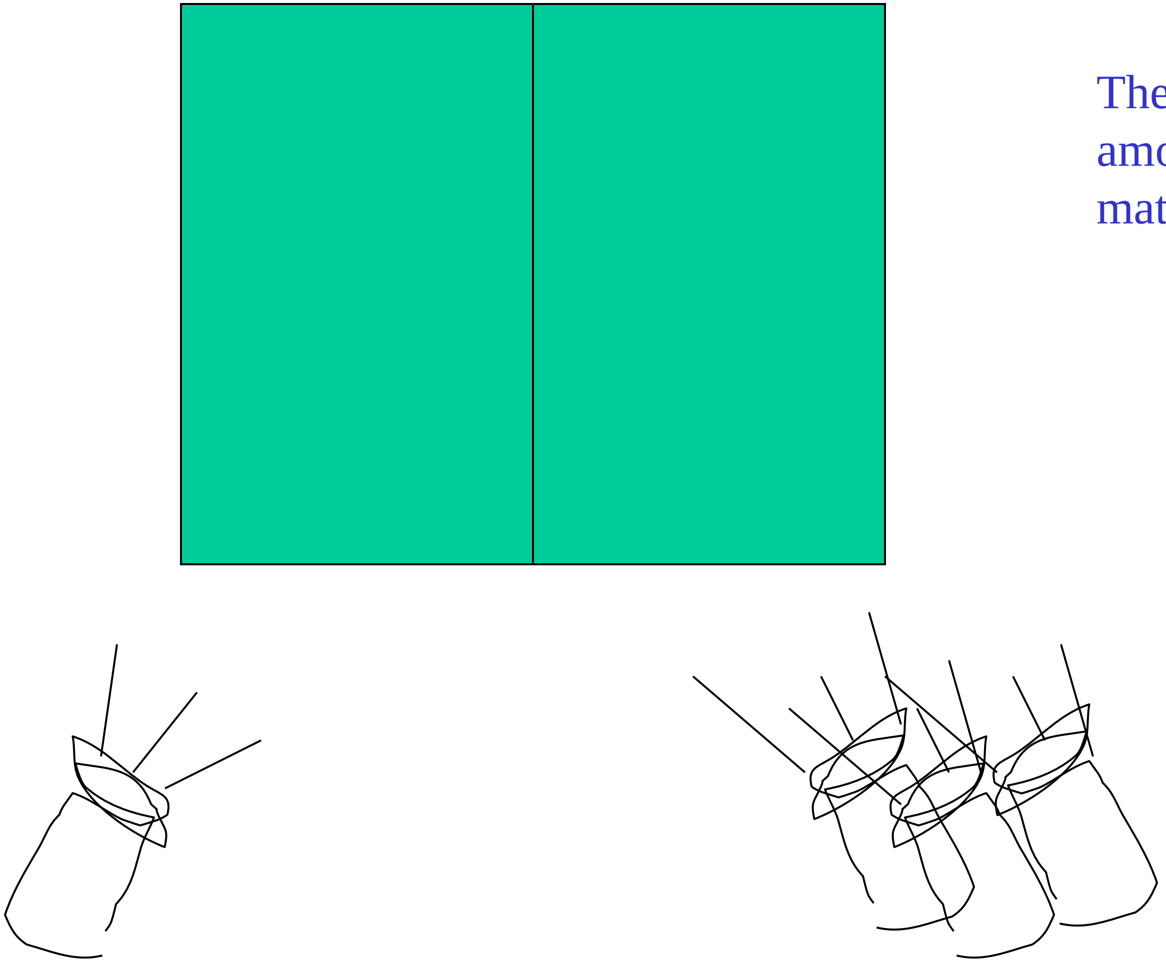
Source: W. Freeman

Color matching experiment 1

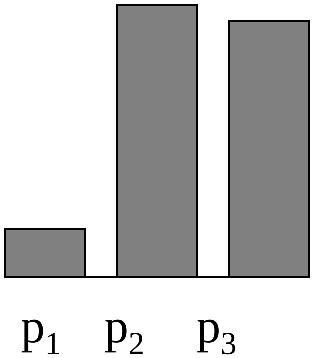


Source: W. Freeman

Color matching experiment 1

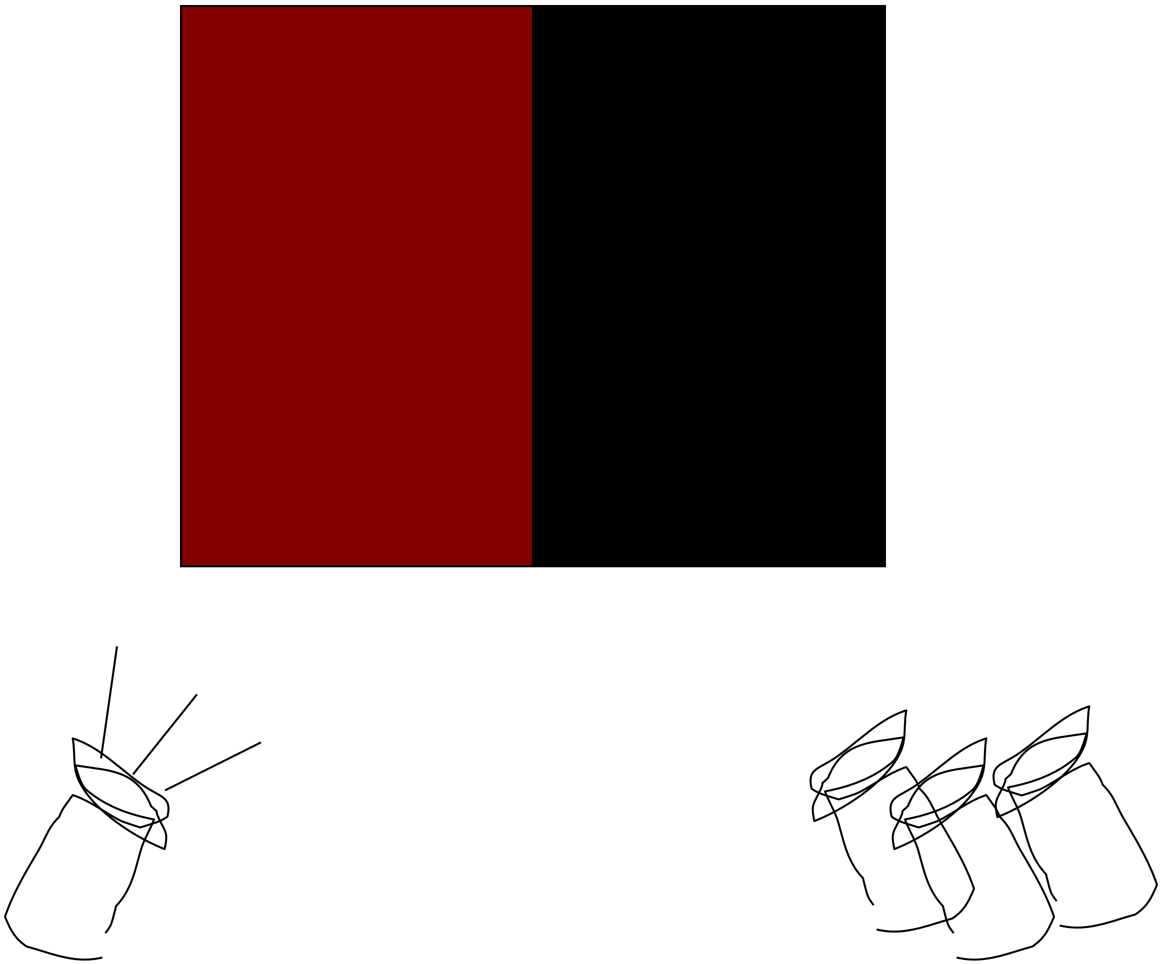


The primary color amounts needed for a match



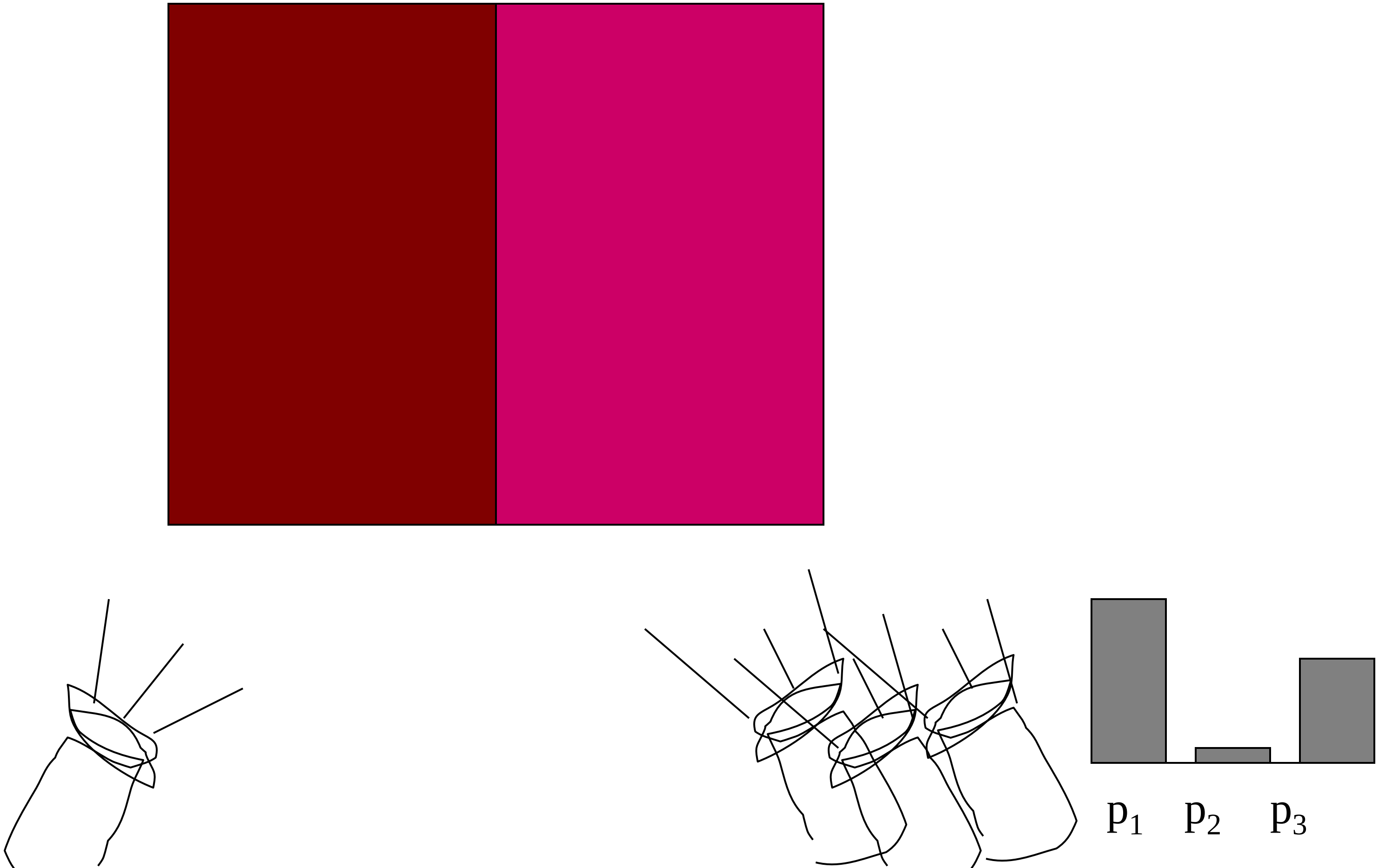
Source: W. Freeman

Color matching experiment 2



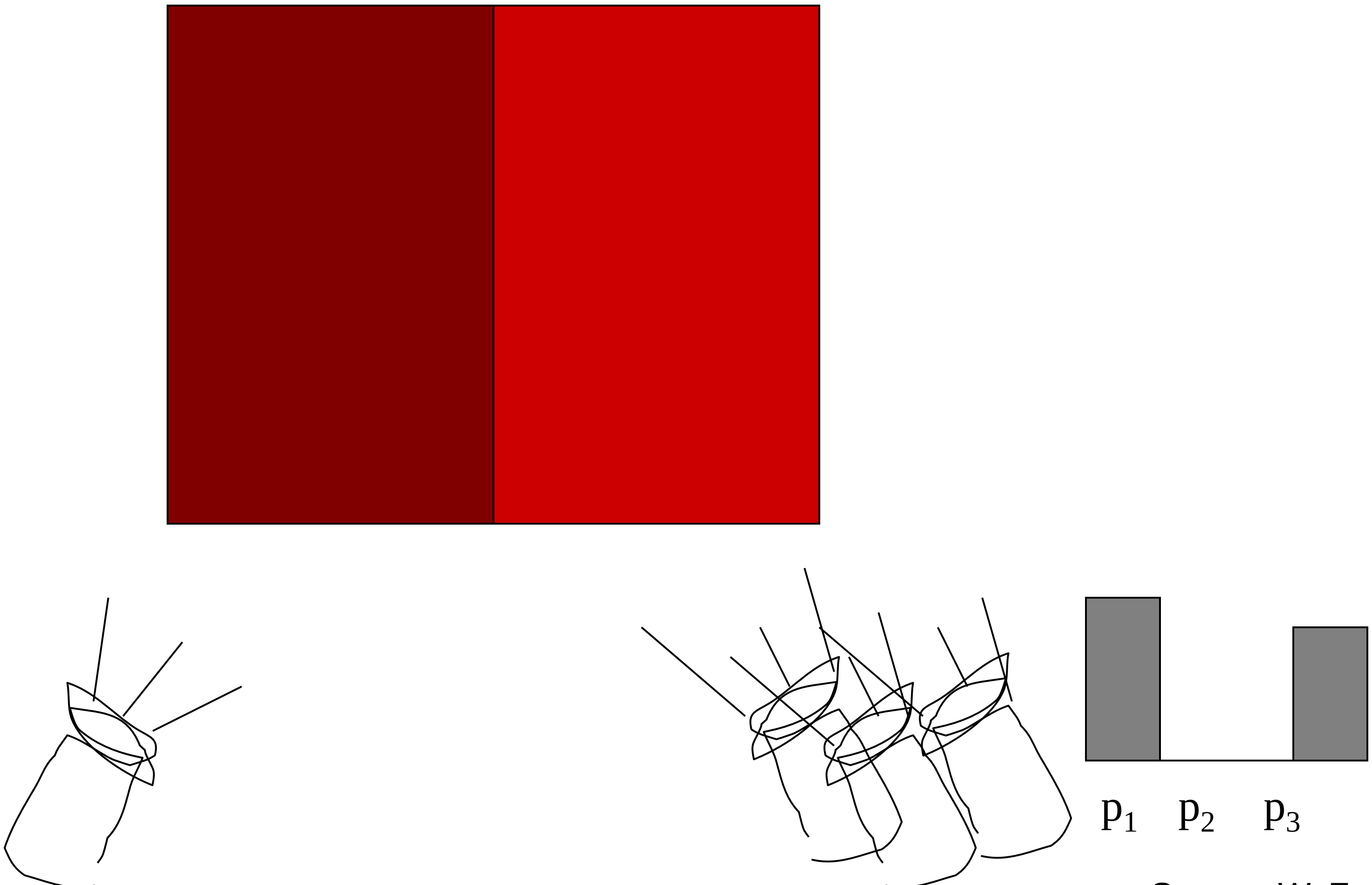
Source: W. Freeman

Color matching experiment 2



Source: W. Freeman

Color matching experiment 2

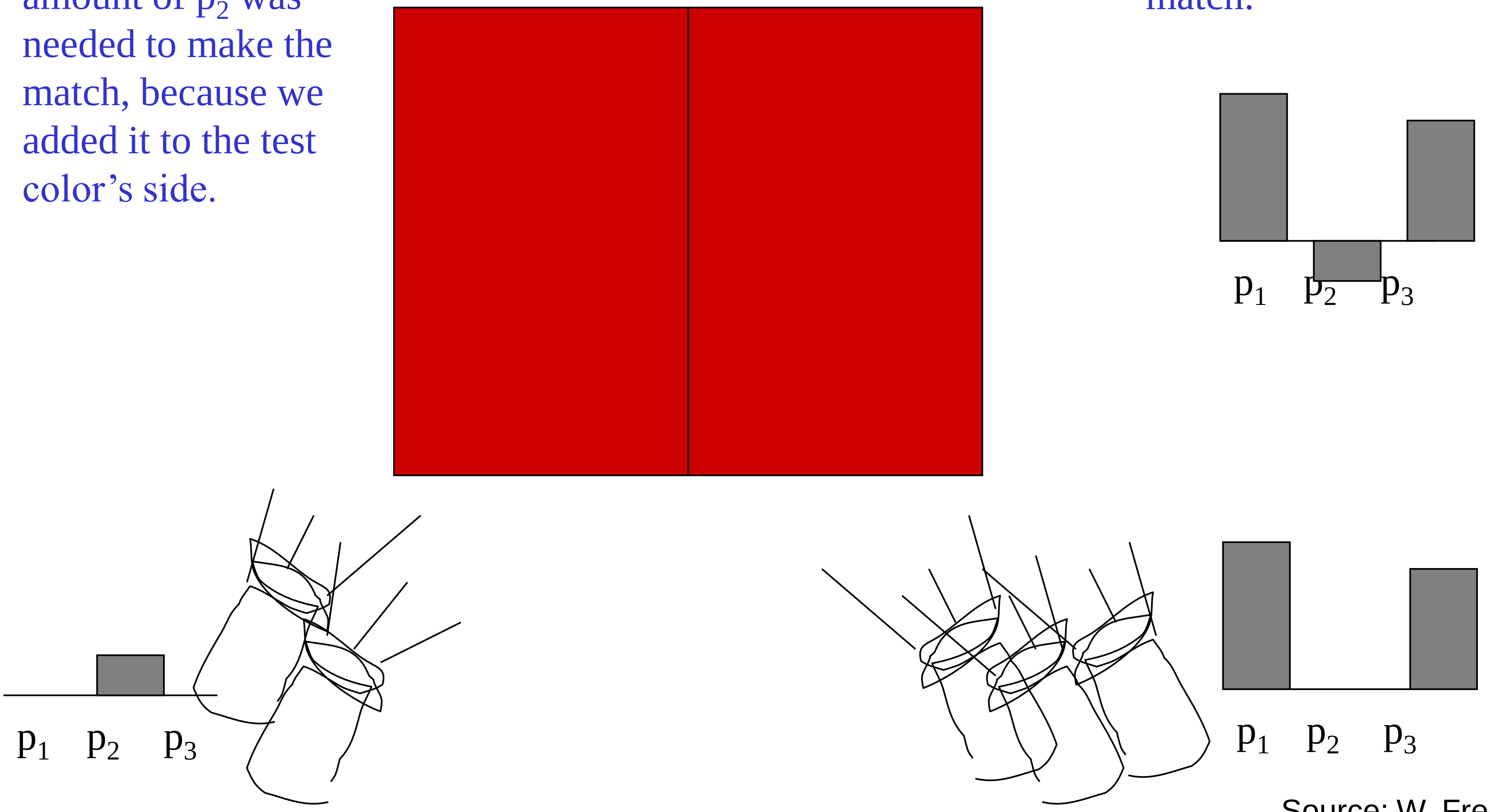


Source: W. Freeman

Color matching experiment 2

We say a "negative" amount of p_2 was needed to make the match, because we added it to the test color's side.

The primary color amounts needed for a match:



Source: W. Freeman

Trichromacy

- In color matching experiments, most people can match any given light with three primaries
  - Primaries must be *independent*
- For the same light and same primaries, most people select the same weights
  - Exception: color blindness
- Trichromatic color theory
  - Three numbers seem to be sufficient for encoding color
  - Dates back to 18th century (Thomas Young)

Grassman's Laws

- Color matching appears to be linear

- If two test lights can be matched with the same set of weights, then they match each other:
  - Suppose $A = u_1\ P_1 + u_2\ P_2 + u_3\ P_3$ and $B = u_1\ P_1 + u_2\ P_2 + u_3\ P_3$. Then $A = B$.

- If we mix two test lights, then mixing the matches will match the result:
  - Suppose $A = u_1\ P_1 + u_2\ P_2 + u_3\ P_3$ and $B = v_1\ P_1 + v_2\ P_2 + v_3\ P_3$. Then $A+B = (u_1+v_1)\ P_1 + (u_2+v_2)\ P_2 + (u_3+v_3)\ P_3$.

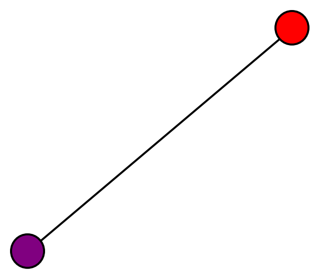
- If we scale the test light, then the matches get scaled by the same amount:
  - Suppose $A = u_1\ P_1 + u_2\ P_2 + u_3\ P_3$. Then $kA = (ku_1)\ P_1 + (ku_2)\ P_2 + (ku_3)\ P_3$.

Overview of Color

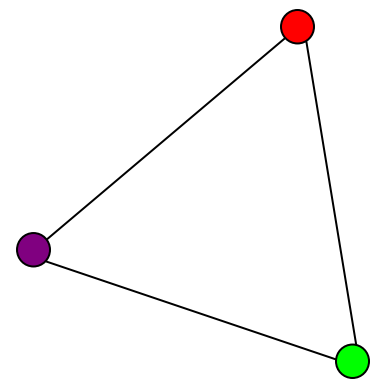
- Physics of color
- Human encoding of color
- Color spaces
- White balancing

Linear color spaces

- Defined by a choice of three *primaries*
- The coordinates of a color are given by the weights of the primaries used to match it

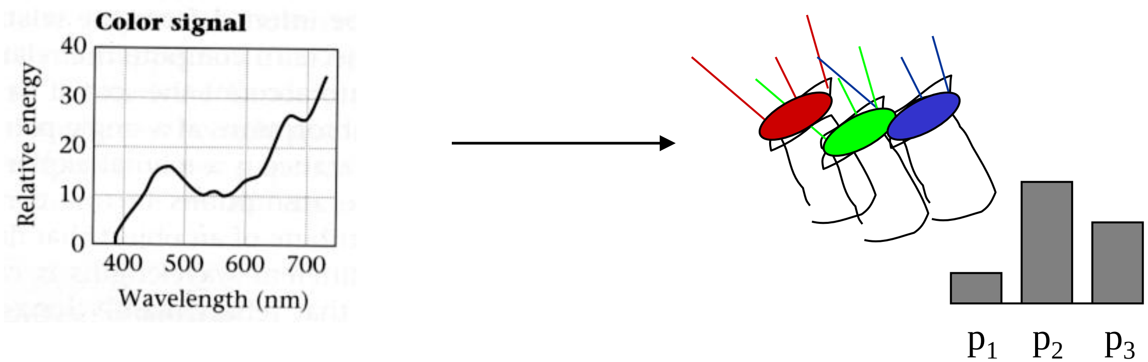


mixing two lights produces colors that lie along a straight line in color space



mixing three lights produces colors that lie within the triangle they define in color space

How to compute the weights of the primaries to match any spectral signal



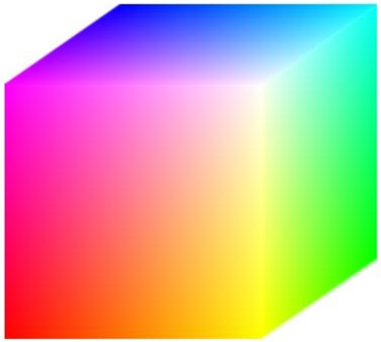
- **Matching functions:** the amount of each primary needed to match a monochromatic light source at each wavelength

Source: W. Freeman

RGB space

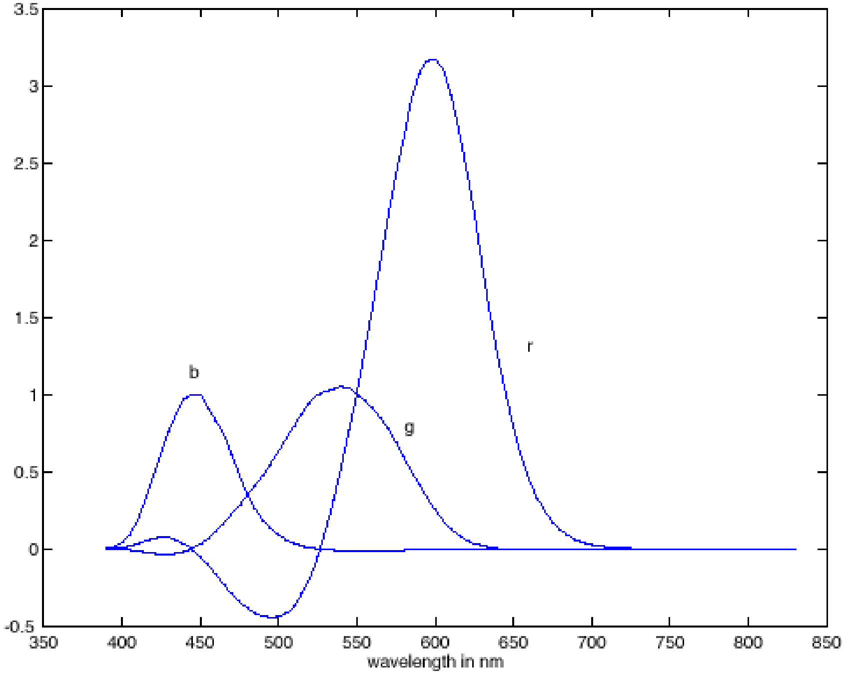
- Primaries are monochromatic lights (for monitors, they correspond to the three types of phosphors)
- *Subtractive matching* required for some wavelengths

RGB primaries



$p_1 = 645.2$ nm
$p_2 = 525.3$ nm
$p_3 = 444.4$ nm

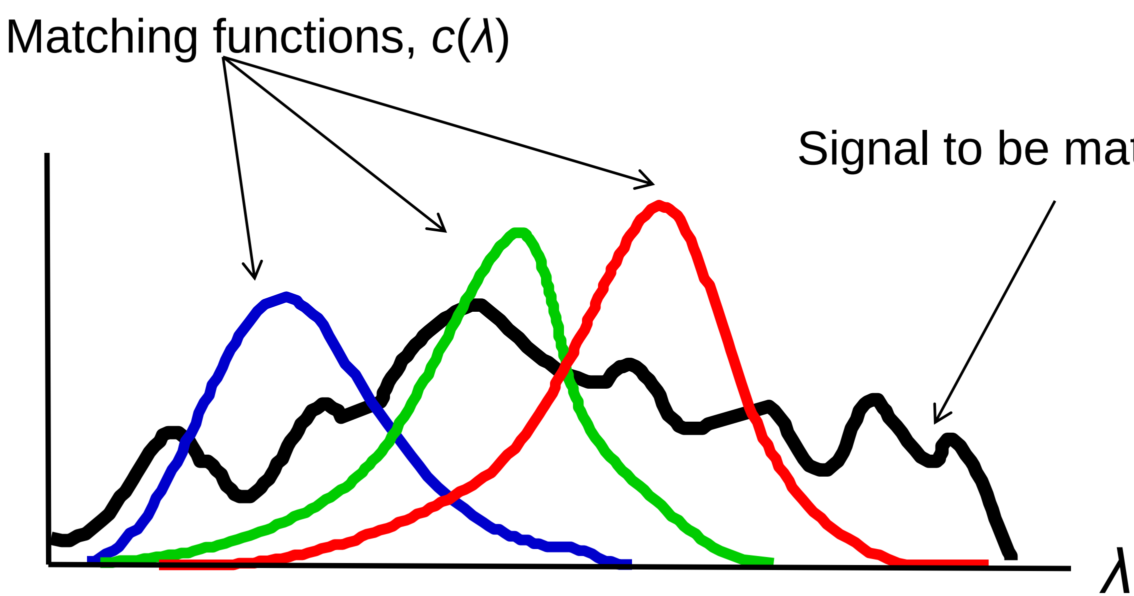
RGB matching functions



How to compute the weights of the primaries to match any spectral signal

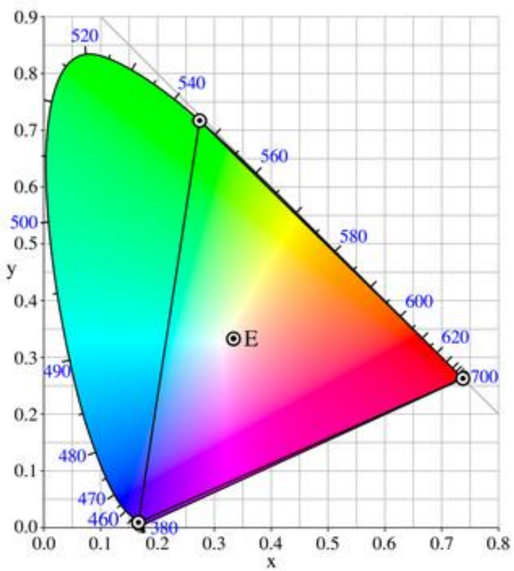
- Let $c(\lambda)$ be one of the matching functions, and let $t(\lambda)$ be the spectrum of the signal. Then the weight of the corresponding primary needed to match t is

$$w = \int_{\lambda} c(\lambda) t(\lambda) d\lambda$$

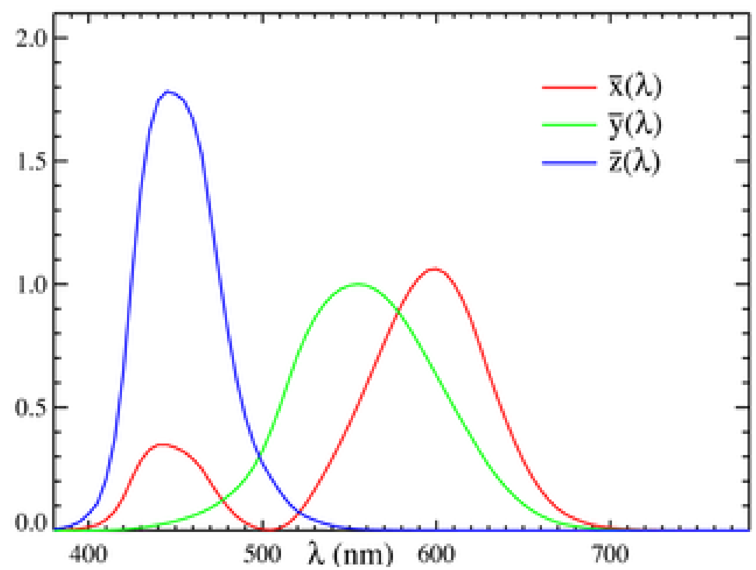


Linear color spaces: CIE XYZ

- Primaries are imaginary, but matching functions are everywhere positive
- The Y parameter corresponds to brightness or *luminance* of a color
- 2D visualization: draw (x,y), where $x = X/(X+Y+Z)$, $y = Y/(X+Y+Z)$

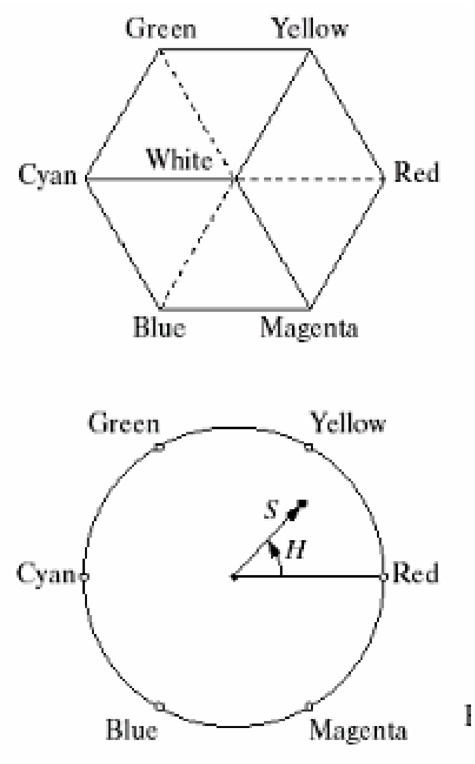


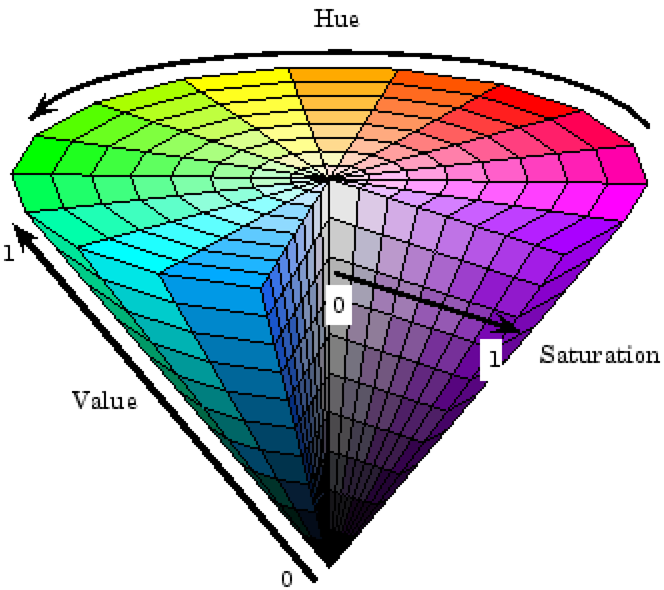
Matching functions

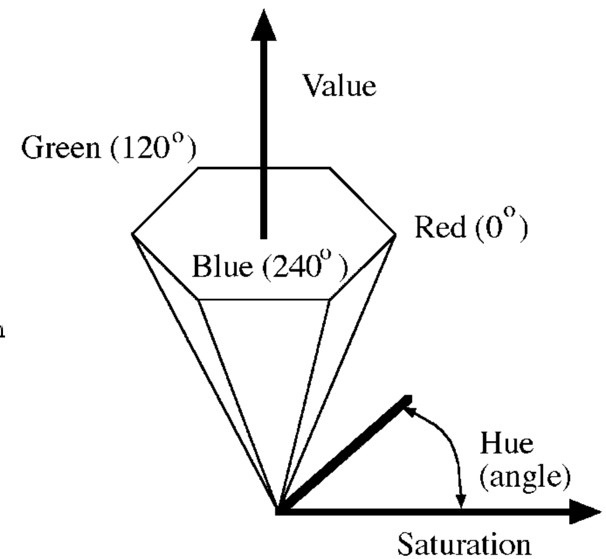


http://en.wikipedia.org/wiki/CIE_1931_color_space

Nonlinear color spaces: HSV



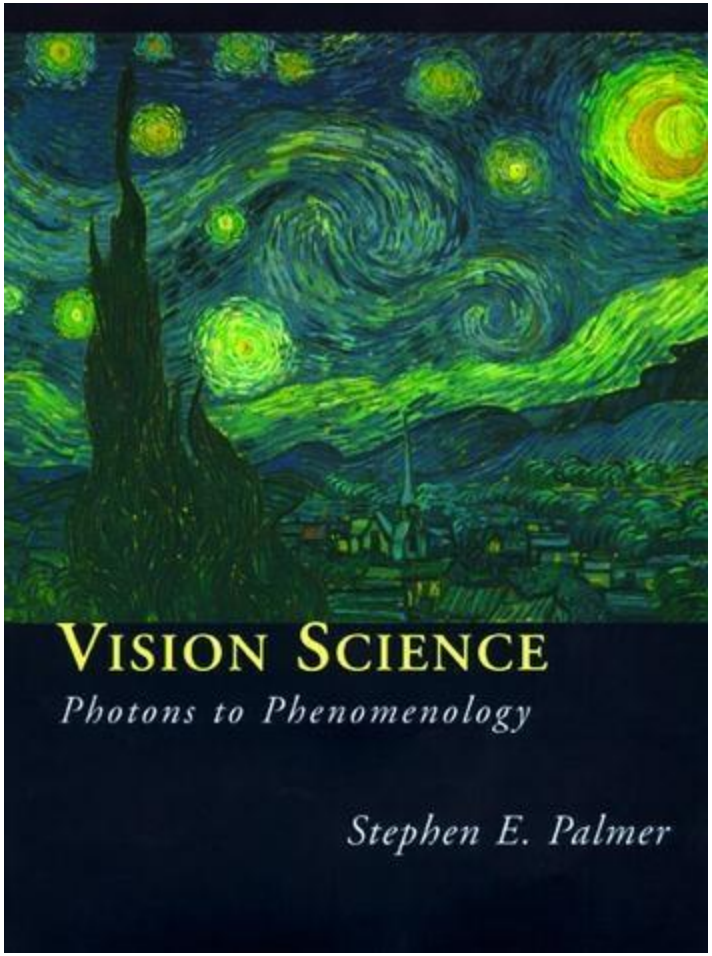




- Perceptually meaningful dimensions: Hue, Saturation, Value (Intensity)
- RGB cube on its vertex

Useful reference

Stephen E. Palmer, **Vision Science: Photons to Phenomenology**, MIT Press, 1999



Overview of Color

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White balance

- When looking at a picture on screen or print, we adapt to the illuminant of the room, not to that of the scene in the picture
- When the white balance is not correct, the picture will have an unnatural color "cast"

incorrect white balance

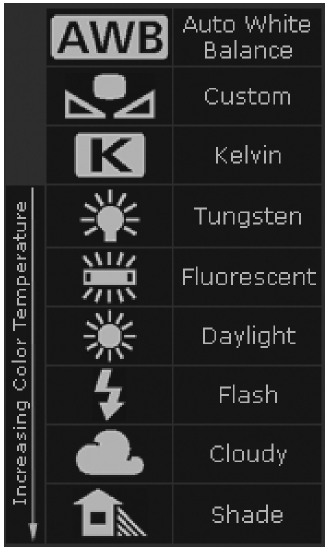
correct white balance

http://www.cambridgeincolour.com/tutorials/white-balance.htm

White balance

- Film cameras:
  - Different types of film or different filters for different illumination conditions

- Digital cameras:
  - Automatic white balance
  - White balance settings corresponding to several common illuminants
  - Custom white balance using a reference object



http://www.cambridgeincolour.com/tutorials/white-balance.htm

Slide: F. Durand

White balance

- Von Kries adaptation
  - Multiply each channel by a gain factor
  - A more general transformation would correspond to an arbitrary 3x3 matrix

Slide: F. Durand

White balance

- Von Kries adaptation
  - Multiply each channel by a gain factor
  - A more general transformation would correspond to an arbitrary 3x3 matrix
- Best way: gray card
  - Take a picture of a neutral object (white or gray)
  - Deduce the weight of each channel
    - If the object is recoded as r_w, g_w, b_w
      use weights $1/r_w$, $1/g_w$, $1/b_w$

Slide: F. Durand

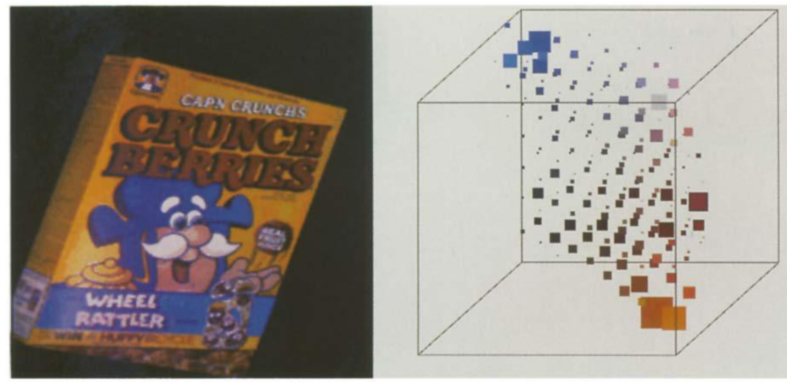
White balance

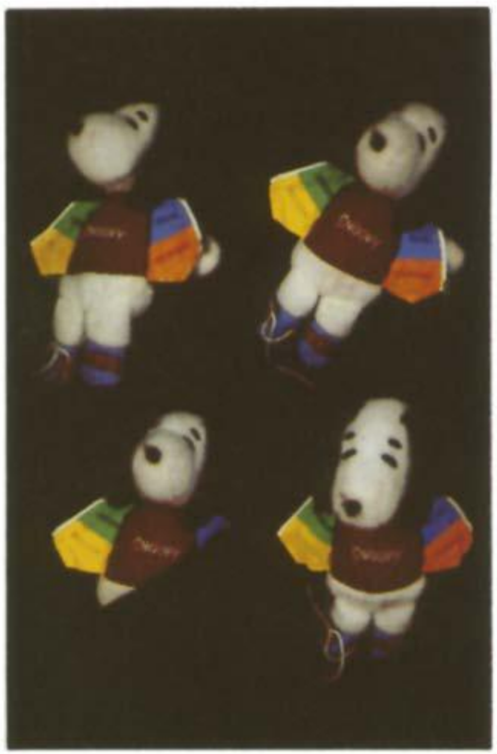
- Without gray cards: we need to “guess” which pixels correspond to white objects
- Gray world assumption
  - The image average r_{ave}, g_{ave}, b_{ave} is gray
  - Use weights $1/r_{ave}$, $1/g_{ave}$, $1/b_{ave}$
- Brightest pixel assumption (non-staurated)
  - Highlights usually have the color of the light source
  - Use weights inversely proportional to the values of the brightest pixels
- Gamut mapping
  - Gamut: convex hull of all pixel colors in an image
  - Find the transformation that matches the gamut of the image to the gamut of a “typical” image under white light
- Use image statistics, learning techniques

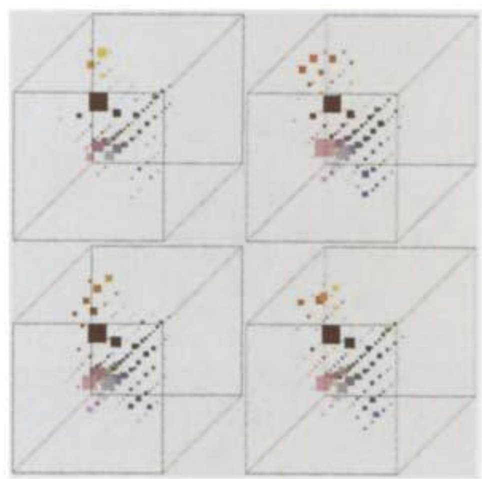
Slide: F. Durand

Uses of color in computer vision

Color histograms for indexing and retrieval



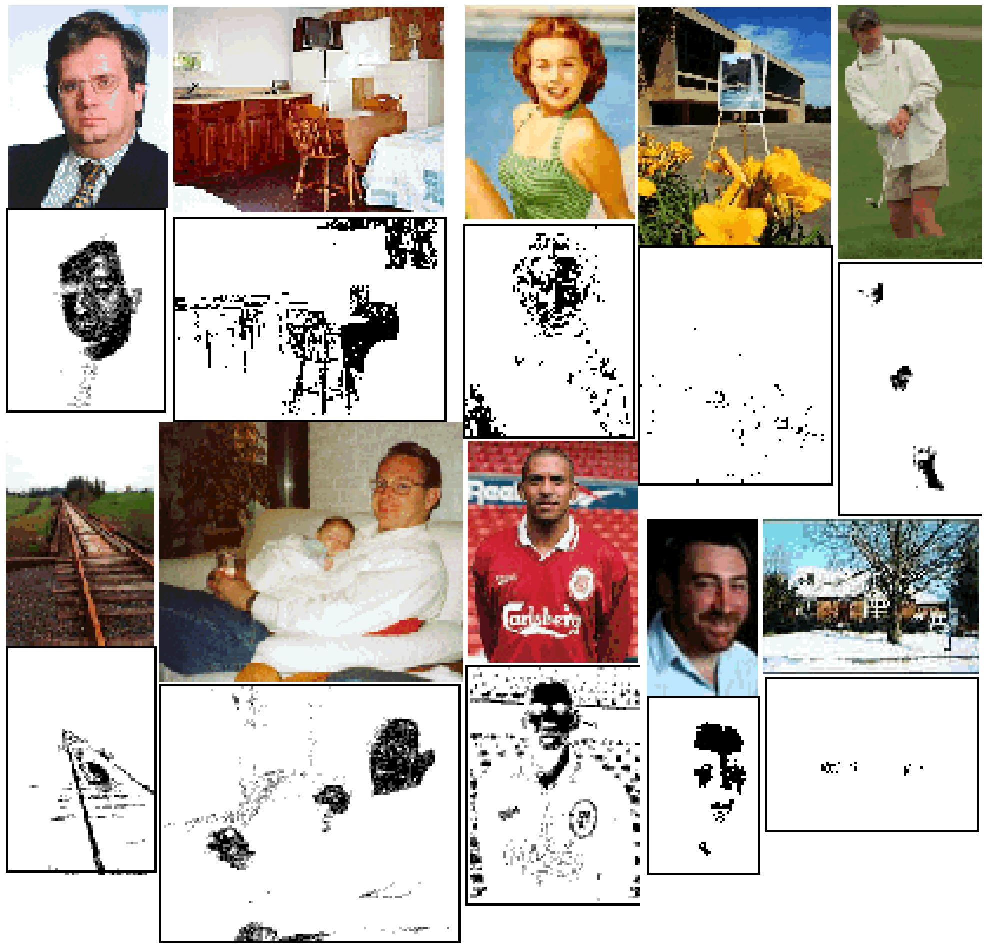




Swain and Ballard, Color Indexing, IJCV 1991.

Uses of color in computer vision

Skin detection

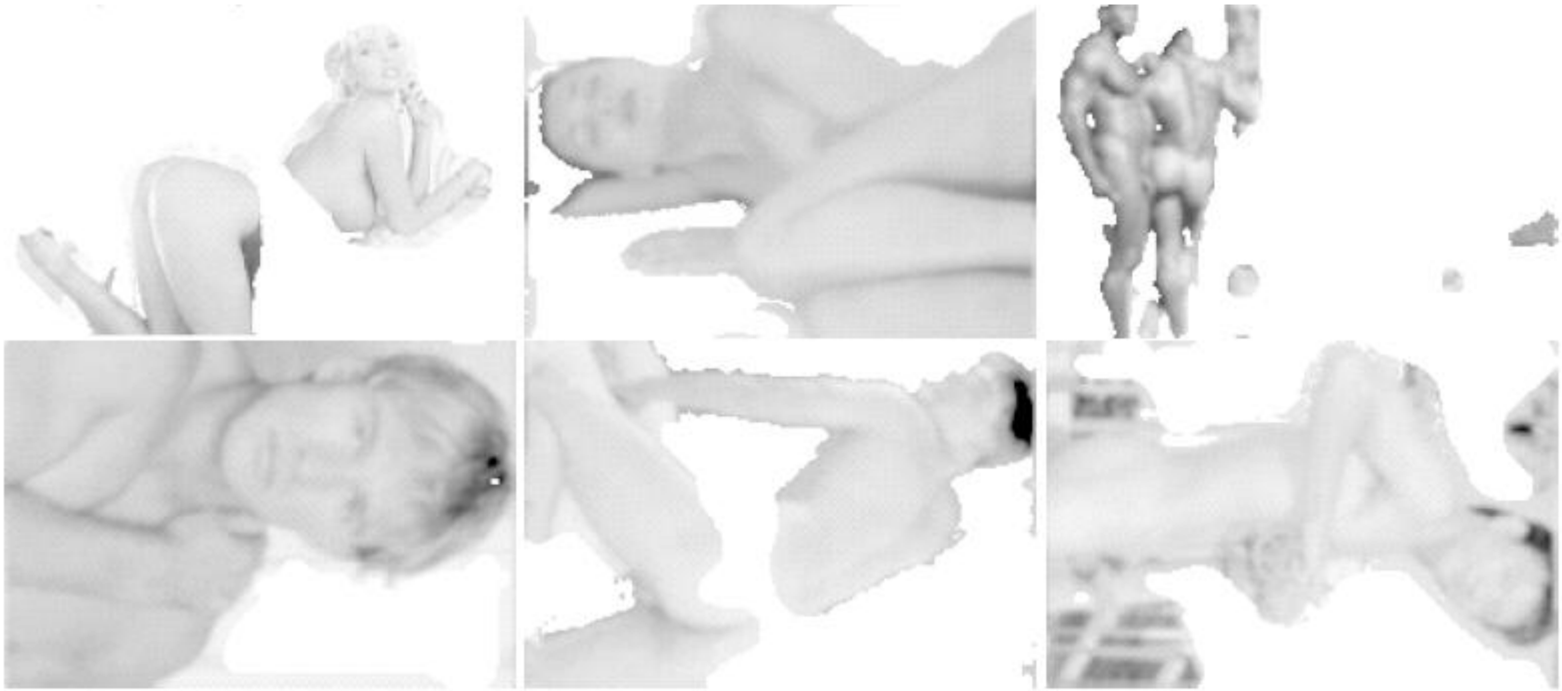


M. Jones and J. Rehg, [Statistical Color Models with Application to Skin Detection](), IJCV 2002.

Source: S. Lazebnik

Uses of color in computer vision

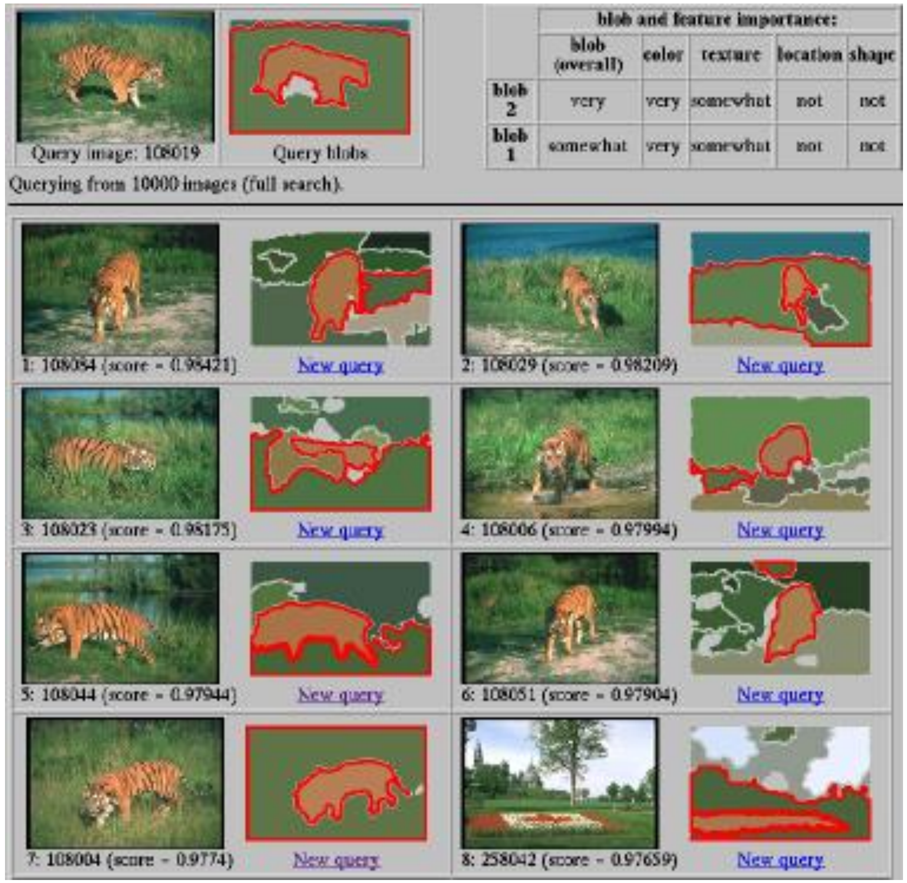
Nude people detection



Forsyth, D.A. and Fleck, M. M., ``Automatic Detection of Human Nudes,'' *International Journal of Computer Vision* , **32** , 1, 63-77, August, 1999

Uses of color in computer vision

Image segmentation and retrieval



C. Carson, S. Belongie, H. Greenspan, and Ji. Malik, Blobworld: Image segmentation using Expectation-Maximization and its application to image querying, ICVIS 1999.

Source: S. Lazebnik

Uses of color in computer vision

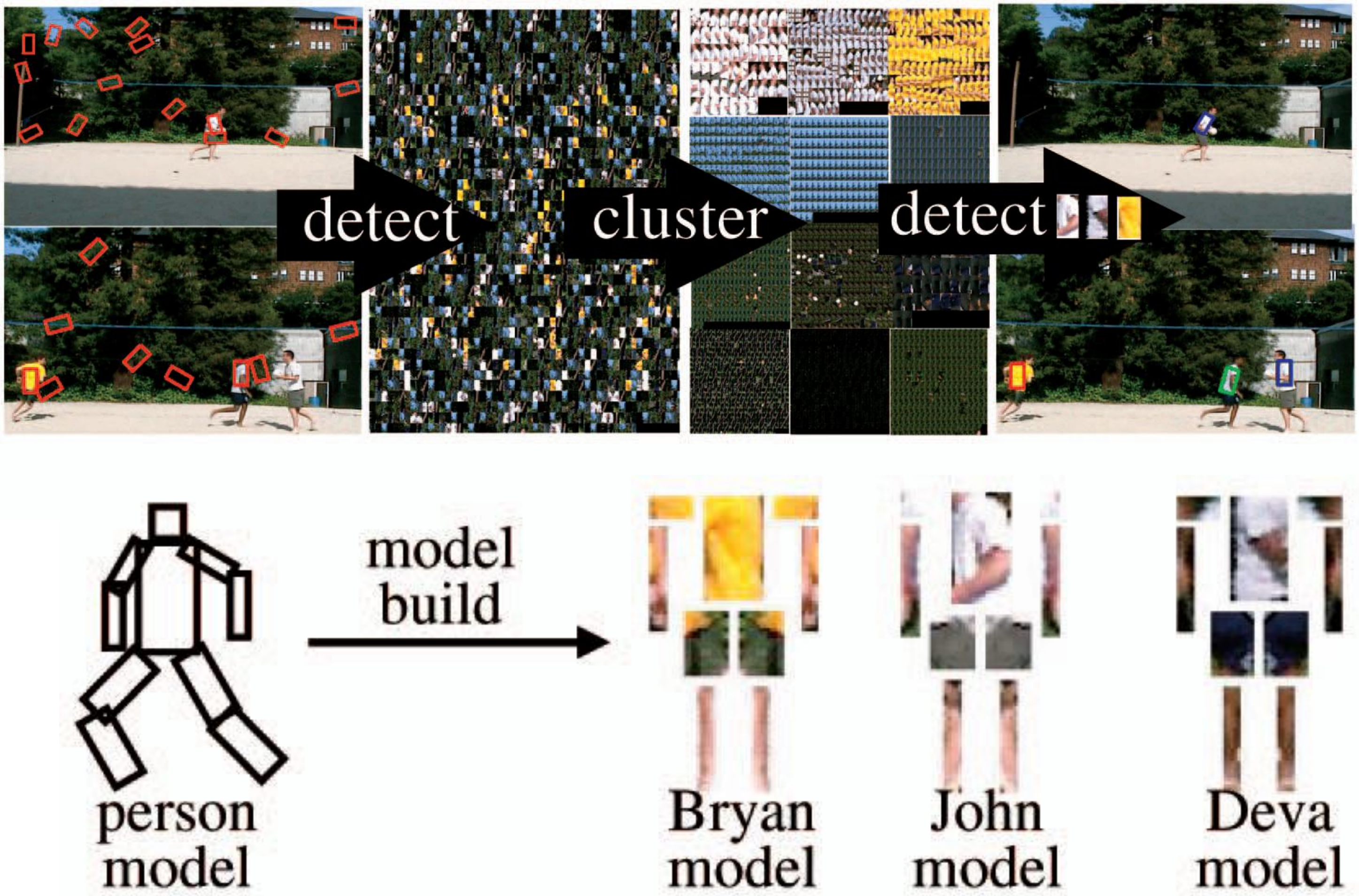
Robot soccer


M. Sridharan and P. Stone, Towards Eliminating Manual Color Calibration at RoboCup. RoboCup-2005: Robot Soccer World Cup IX, Springer Verlag, 2006

Source: K. Grauman

Uses of color in computer vision

Building appearance models for tracking



D. Ramanan, D. Forsyth, and A. Zisserman. [Tracking People by Learning their Appearance](). PAMI 2007.

Source: S. Lazebnik

Interlude

- Next class at 5pm Thursday

- But next week: class at 7pm Tuesday
  - Prof. Bregler will teach it (I am away)

- Back to normal afterwards (5pm Thursday)

Image Filtering

Overview of Filtering

- Convolution
- Gaussian filtering
- Median filtering

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- Convolution
- Gaussian filtering
- Median filtering

Motivation: Noise reduction

- Given a camera and a still scene, how can you reduce noise?

Take lots of images and average them!

What's the next best thing?

Source: S. Seitz

Moving average

- Let's replace each pixel with a *weighted* average of its neighborhood
- The weights are called the *filter kernel*
- What are the weights for the average of a 3x3 neighborhood?

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

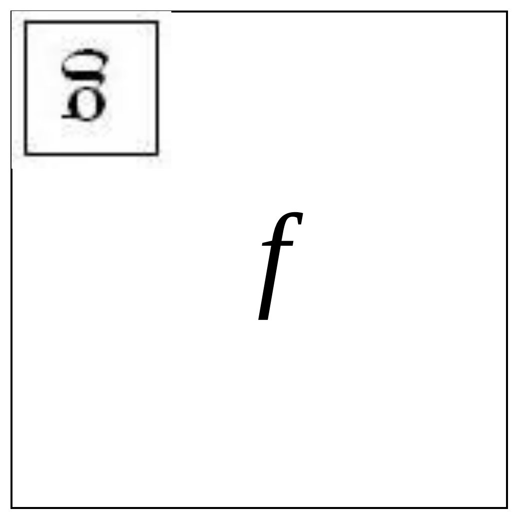
"box filter"

Source: D. Lowe

Defining Convolution

- Let *f* be the image and *g* be the kernel. The output of convolving *f* with *g* is denoted *f* * *g*.

$$(f * g)[m,n] = \sum_{k,l} f[m-k, n-l]\, g[k,l]$$



- Convention: kernel is "flipped"
- MATLAB: conv2 (also imfilter)

Source: F. Durand

Key properties

- **Linearity:** filter($f_1 + f_2$) = filter(f_1) + filter(f_2)

- **Shift invariance:** same behavior regardless of pixel location: filter(shift(f)) = shift(filter(f))

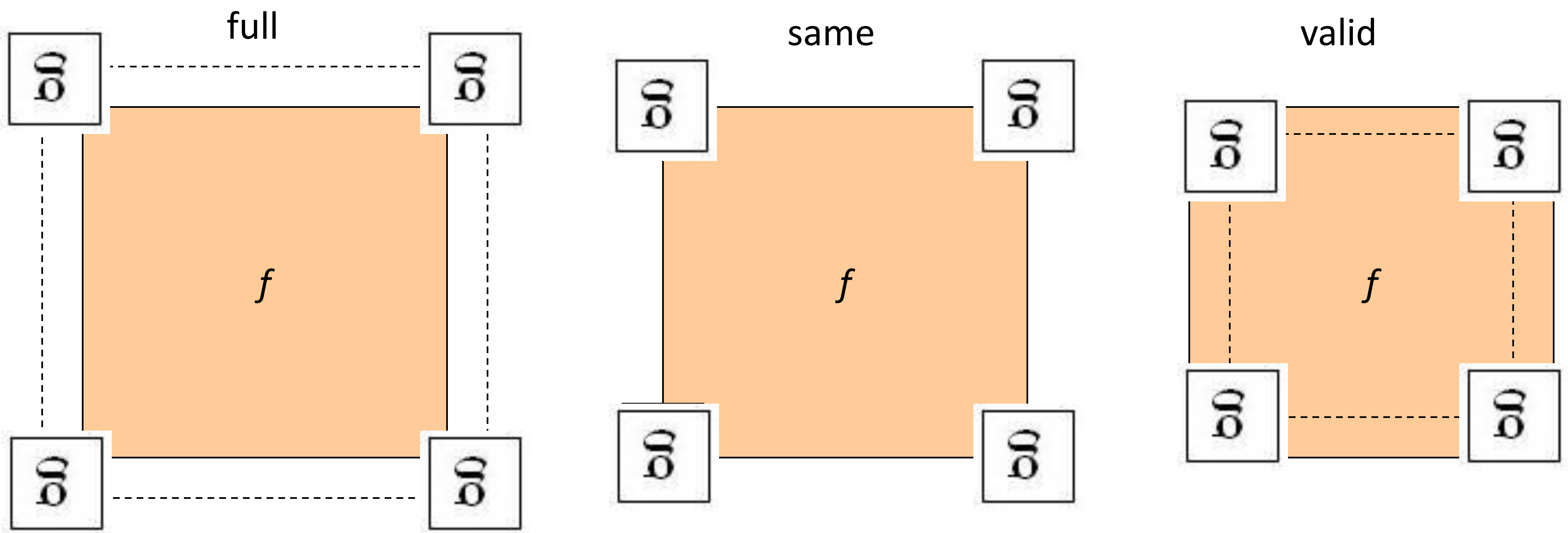
- Theoretical result: any linear shift-invariant operator can be represented as a convolution

Properties in more detail

- Commutative: $a * b = b * a$
  - Conceptually no difference between filter and signal
- Associative: $a * (b * c) = (a * b) * c$
  - Often apply several filters one after another: $(((a * b_1) * b_2) * b_3)$
  - This is equivalent to applying one filter: a * $(b_1 * b_2 * b_3)$
- Distributes over addition: $a * (b + c) = (a * b) + (a * c)$
- Scalars factor out: $ka * b = a * kb = k\,(a * b)$
- Identity: unit impulse $e = [\ldots, 0, 0, 1, 0, 0, \ldots]$, $a * e = a$

Annoying details

- What is the size of the output?
- MATLAB: conv2(f, g,*shape*)
  - *shape* = 'full': output size is sum of sizes of f and g
  - *shape* = 'same': output size is same as f
  - *shape* = 'valid': output size is difference of sizes of f and g



Annoying details

- What about near the edge?
  - the filter window falls off the edge of the image
  - need to extrapolate
  - methods:
    - clip filter (black)
    - wrap around
    - copy edge
    - reflect across edge

Source: S. Marschner

Annoying details

- What about near the edge?
  - the filter window falls off the edge of the image
  - need to extrapolate
  - methods (MATLAB):
    - clip filter (black): imfilter(f, g, 0)
    - wrap around: imfilter(f, g, 'circular')
    - copy edge: imfilter(f, g, 'replicate')
    - reflect across edge: imfilter(f, g, 'symmetric')

Source: S. Marschner

Practice with linear filters

Original

0	0	0
0	1	0
0	0	0

?

Source: D. Lowe

Practice with linear filters

Original

0	0	0
0	1	0
0	0	0

Filtered
(no change)

Source: D. Lowe

Practice with linear filters

Original

0	0	0
0	0	1
0	0	0

?

Source: D. Lowe

Practice with linear filters

Original

0	0	0
0	0	1
0	0	0

Shifted left
By 1 pixel

Source: D. Lowe

Practice with linear filters

Original

$$\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

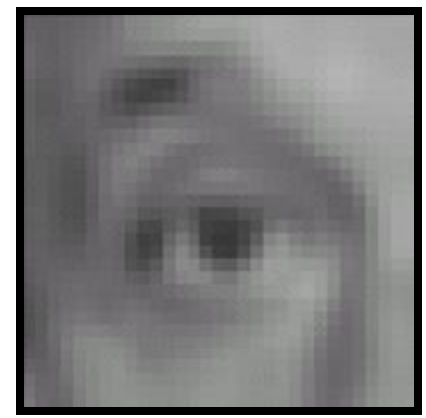
?

Source: D. Lowe

Practice with linear filters

Original

$$\frac{1}{9}\begin{bmatrix}1 & 1 & 1\\1 & 1 & 1\\1 & 1 & 1\end{bmatrix}$$



Blur (with a box filter)

Source: D. Lowe

Practice with linear filters

Original

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \quad ?$$

(Note that filter sums to 1)

Source: D. Lowe

Practice with linear filters

Original

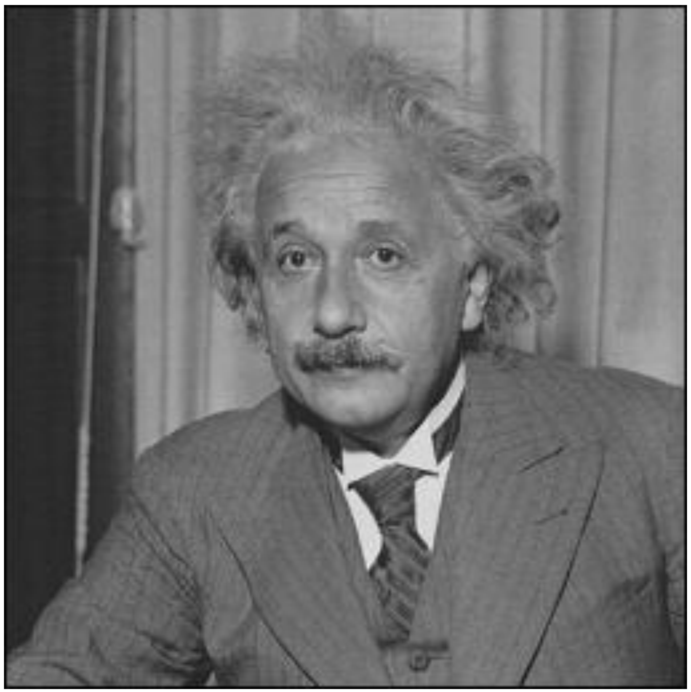
$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Sharpening filter

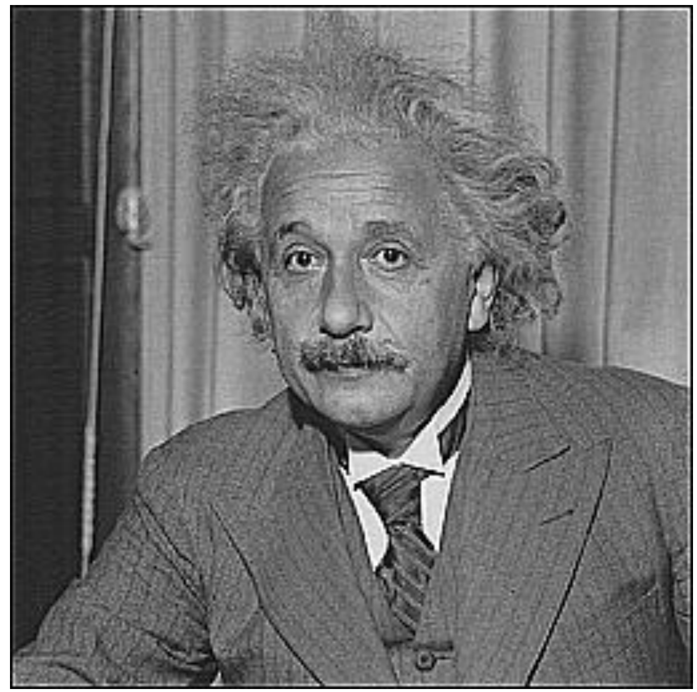
- Accentuates differences with local average

Source: D. Lowe

Sharpening



before



after

Slide credit: Bill Freeman

Spatial resolution and color

original

R

G

B

Slide credit: Bill Freeman

Blurring the G component

original

processed

R

G

B

Slide credit: Bill Freeman

Blurring the R component

original

processed

R

G

B

Slide credit: Bill Freeman

Blurring the B component

original

processed

R

G

B

Slide credit: Bill Freeman

From W. E. Glenn, in Digital Images and Human Vision, MIT Press, edited by Watson, 1993

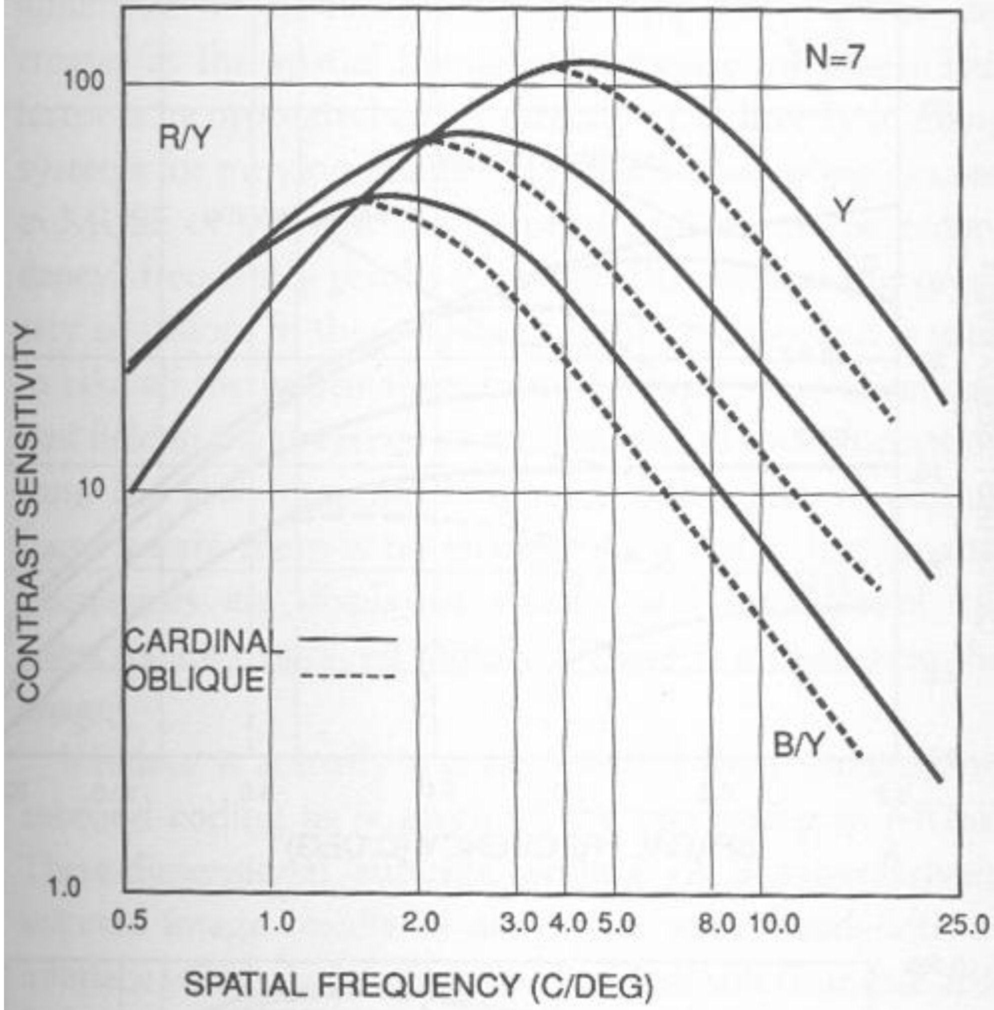


Figure 6.1

Contrast sensitivity threshold functions for static luminance gratings (Y) and isoluminance chromaticity gratings (R/Y,B/Y) averaged over seven observers.

Slide credit: Bill Freeman

Lab color components

L

a

b

A rotation of the color coordinates into directions that are more perceptually meaningful:
L: luminance,
a: red-green,
b: blue-yellow

Slide credit: Bill Freeman

Blurring the L Lab component

original

processed

L

a

b

Slide credit: Bill Freeman

Blurring the a Lab component

original

processed

L

a

b

Slide credit: Bill Freeman

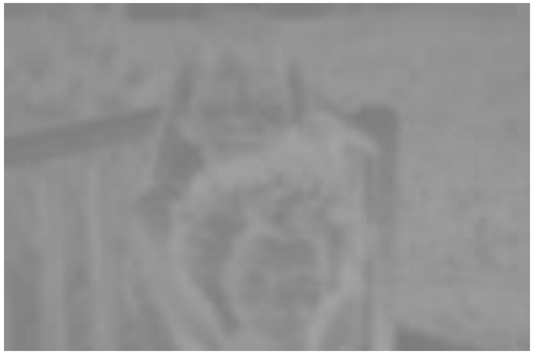
Blurring the b Lab component

original

processed

L

a



b

Slide credit: Bill Freeman

Overview of Filtering

- Convolution
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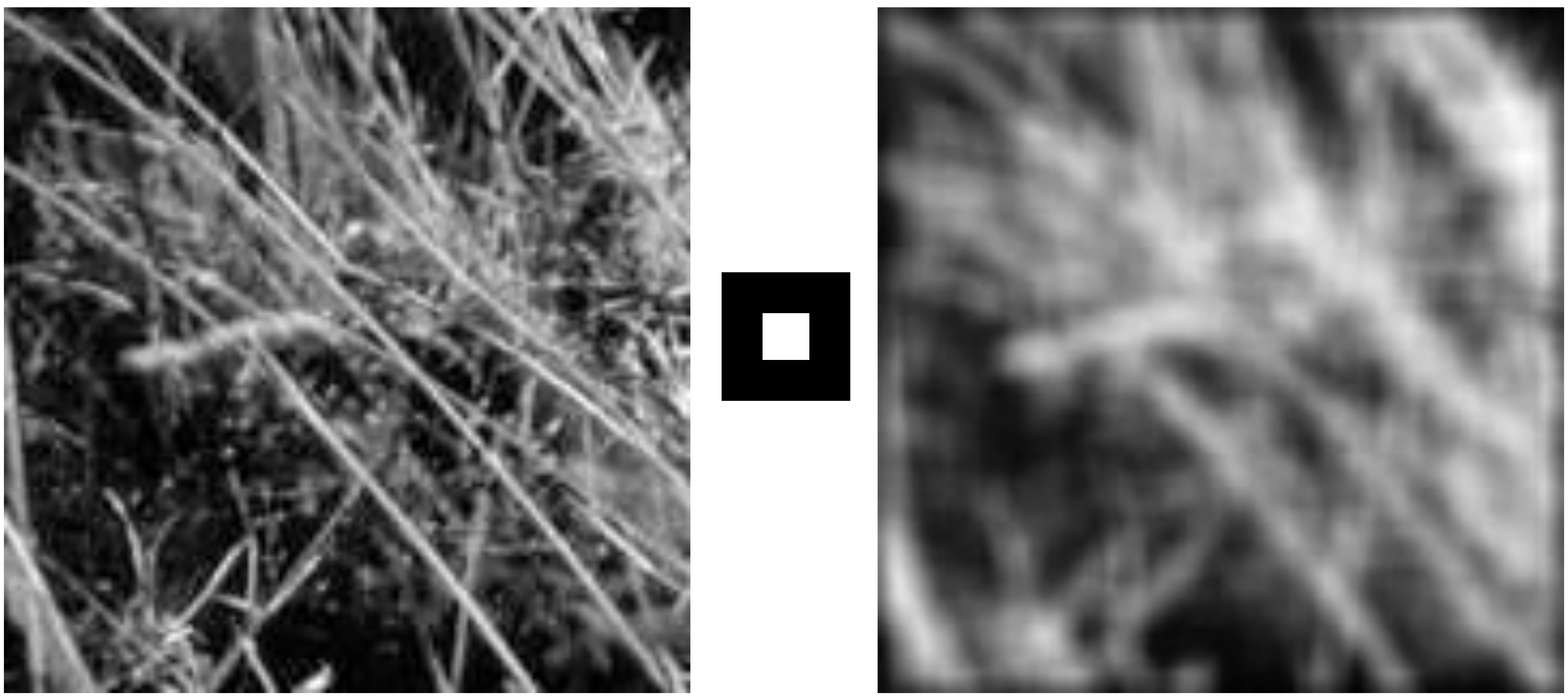
Smoothing with box filter revisited

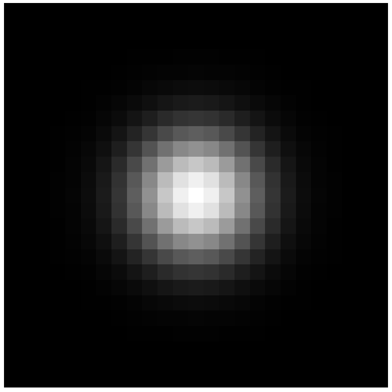
- Smoothing with an average actually doesn't compare at all well with a defocused lens
- Most obvious difference is that a single point of light viewed in a defocused lens looks like a fuzzy blob; but the averaging process would give a little square



Source: D. Forsyth

Smoothing with box filter revisited

- Smoothing with an average actually doesn't compare at all well with a defocused lens
- Most obvious difference is that a single point of light viewed in a defocused lens looks like a fuzzy blob; but the averaging process would give a little square
- Better idea: to eliminate edge effects, weight contribution of neighborhood pixels according to their closeness to the center, like so:

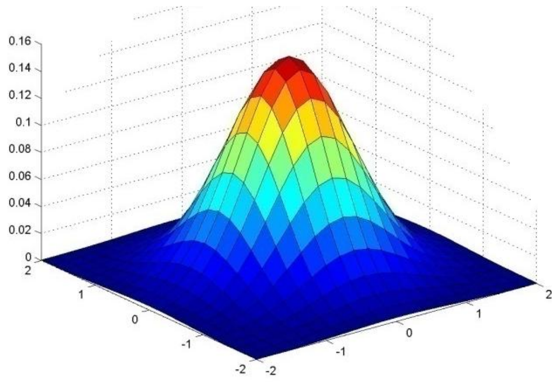


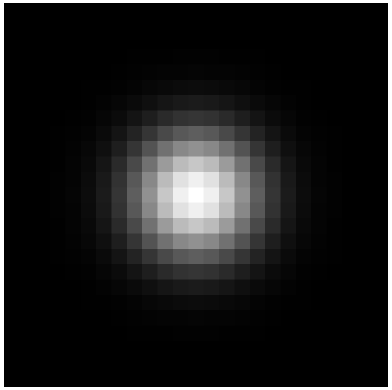
"fuzzy blob"

Source: D. Forsyth

Gaussian Kernel

$$G_\sigma = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$





0.003	0.013	0.022	0.013	0.003
0.013	0.059	0.097	0.059	0.013
0.022	0.097	0.159	0.097	0.022
0.013	0.059	0.097	0.059	0.013
0.003	0.013	0.022	0.013	0.003

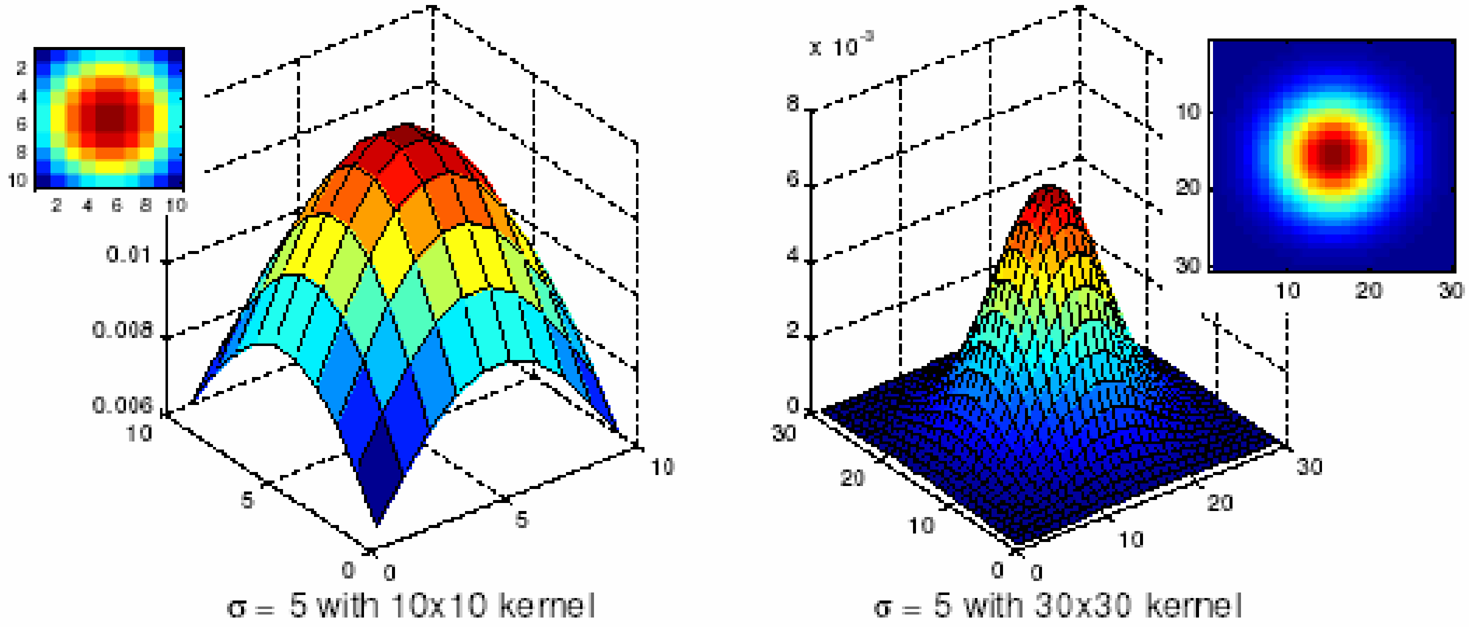
5 x 5, σ = 1

- Constant factor at front makes volume sum to 1 (can be ignored, as we should re-normalize weights to sum to 1 in any case)

Source: C. Rasmussen

Choosing kernel width

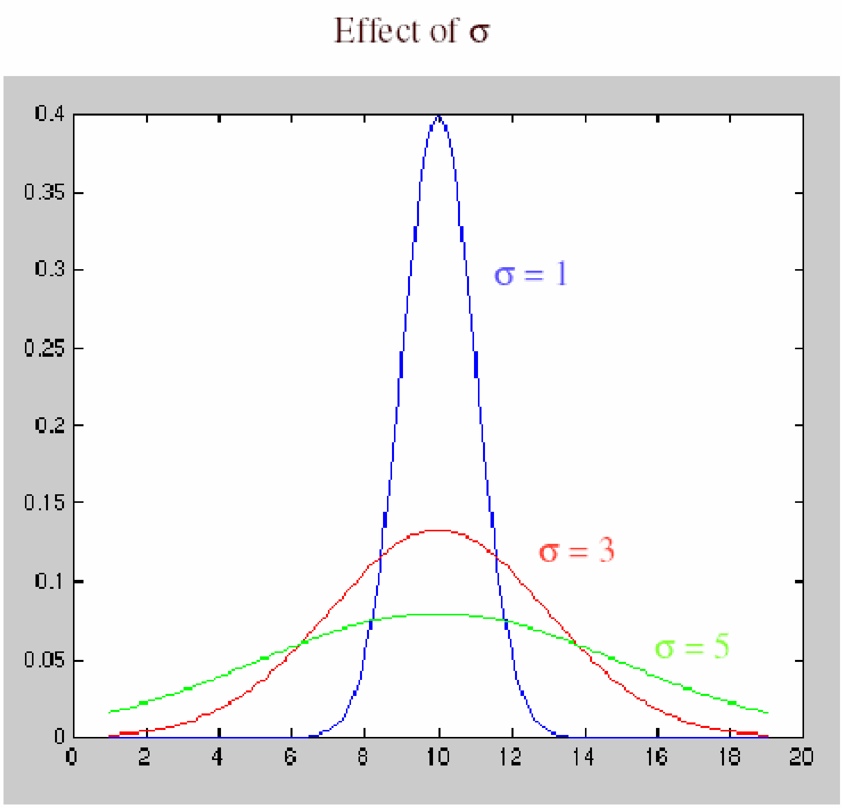
- Gaussian filters have infinite support, but discrete filters use finite kernels



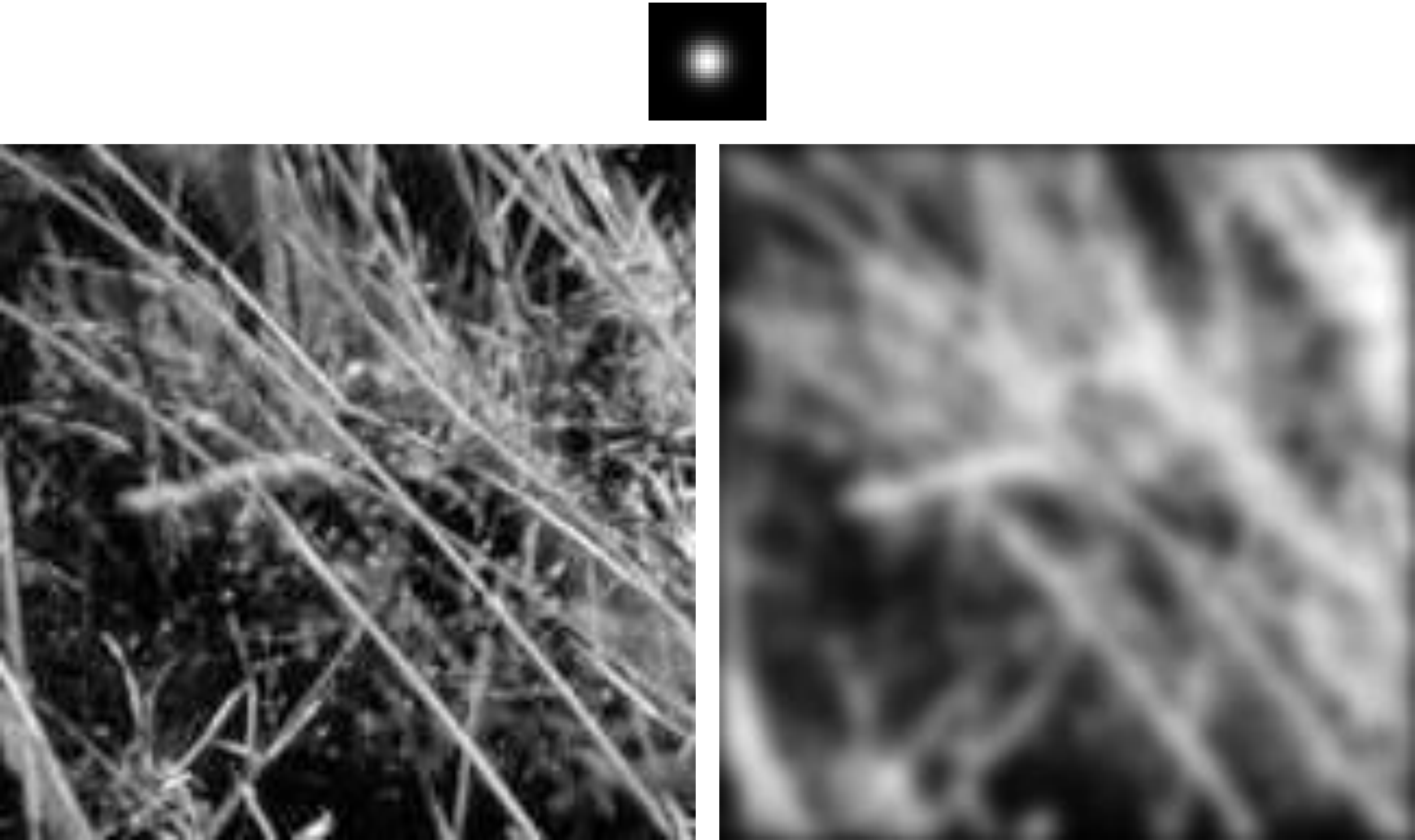
Source: K. Grauman

Choosing kernel width

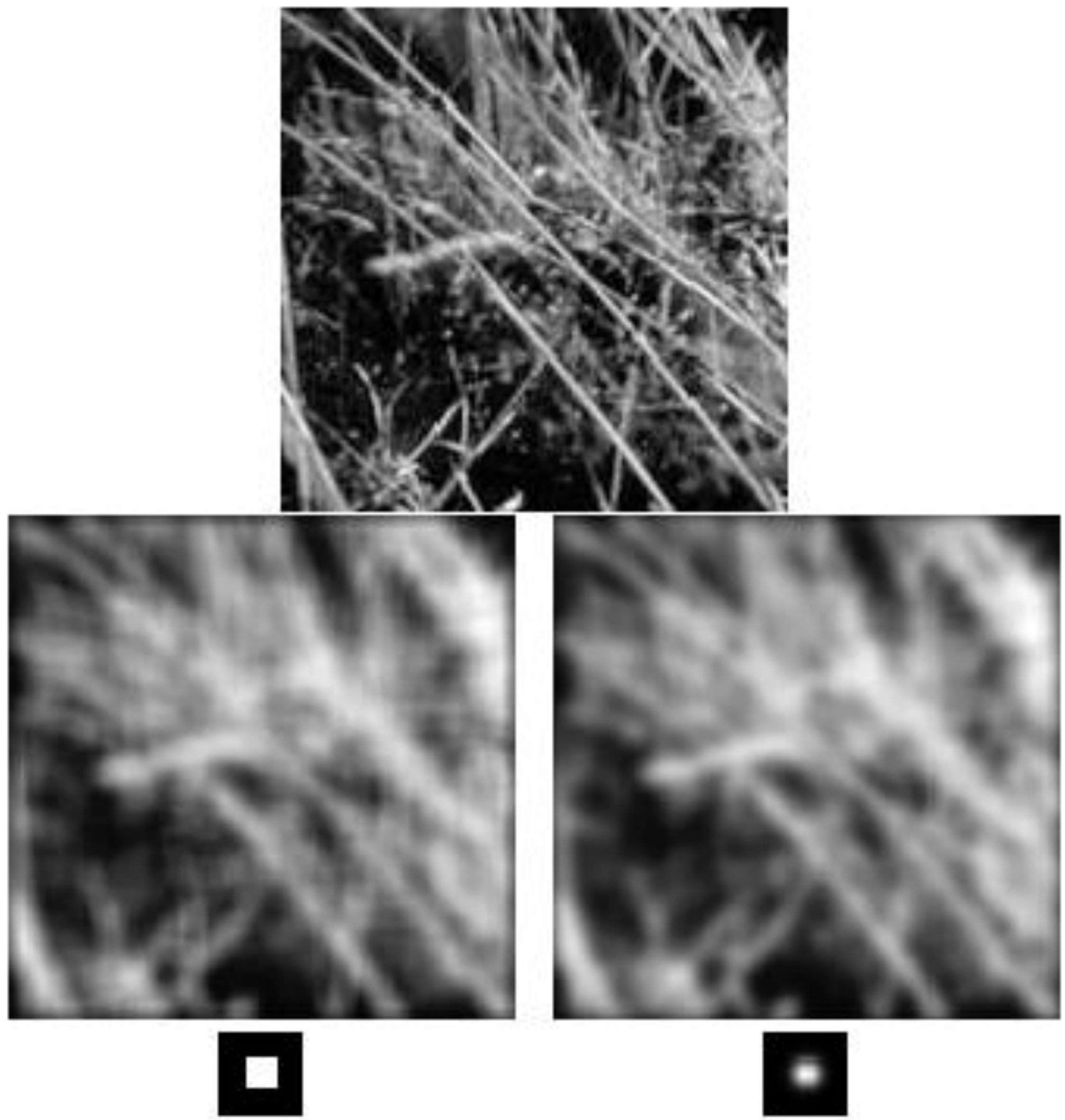
- Rule of thumb: set filter half-width to about 3 σ



Example: Smoothing with a Gaussian



Mean vs. Gaussian filtering



Gaussian filters

- Remove “high-frequency” components from the image (low-pass filter)
- Convolution with self is another Gaussian
  - So can smooth with small-width kernel, repeat, and get same result as larger-width kernel would have
  - Convolving two times with Gaussian kernel of width σ is same as convolving once with kernel of width $\sigma\sqrt{2}$
- *Separable* kernel
  - Factors into product of two 1D Gaussians

Source: K. Grauman

Separability of the Gaussian filter

$$G_\sigma(x, y) = \frac{1}{2\pi\sigma^2} \exp^{-\frac{x^2+y^2}{2\sigma^2}}$$

$$= \left(\frac{1}{\sqrt{2\pi}\sigma} \exp^{-\frac{x^2}{2\sigma^2}}\right) \left(\frac{1}{\sqrt{2\pi}\sigma} \exp^{-\frac{y^2}{2\sigma^2}}\right)$$

The 2D Gaussian can be expressed as the product of two functions, one a function of x and the other a function of y

In this case, the two functions are the (identical) 1D Gaussian

Source: D. Lowe

Separability example

2D convolution
(center location only)

$$\begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array} * \begin{array}{|c|c|c|}\hline 2 & 3 & 3 \\ \hline 3 & 5 & 5 \\ \hline 4 & 4 & 6 \\ \hline \end{array}$$

The filter factors
into a product of 1D
filters:

$$\begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline 2 & 4 & 2 \\ \hline 1 & 2 & 1 \\ \hline \end{array} = \begin{array}{|c|}\hline 1 \\ \hline 2 \\ \hline 1 \\ \hline \end{array} \times \begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline \end{array}$$

Perform convolution
along rows:

$$\begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline \end{array} * \begin{array}{|c|c|c|}\hline 2 & 3 & 3 \\ \hline 3 & 5 & 5 \\ \hline 4 & 4 & 6 \\ \hline \end{array} = \begin{array}{|c|c|c|}\hline & 11 & \\ \hline & 18 & \\ \hline & 18 & \\ \hline \end{array}$$

Followed by convolution
along the remaining column:

**For MN image, PQ filter: 2D takes MNPQ add/times,
while 1D takes MN(P + Q)**

Source: K. Grauman

Overview of Filtering

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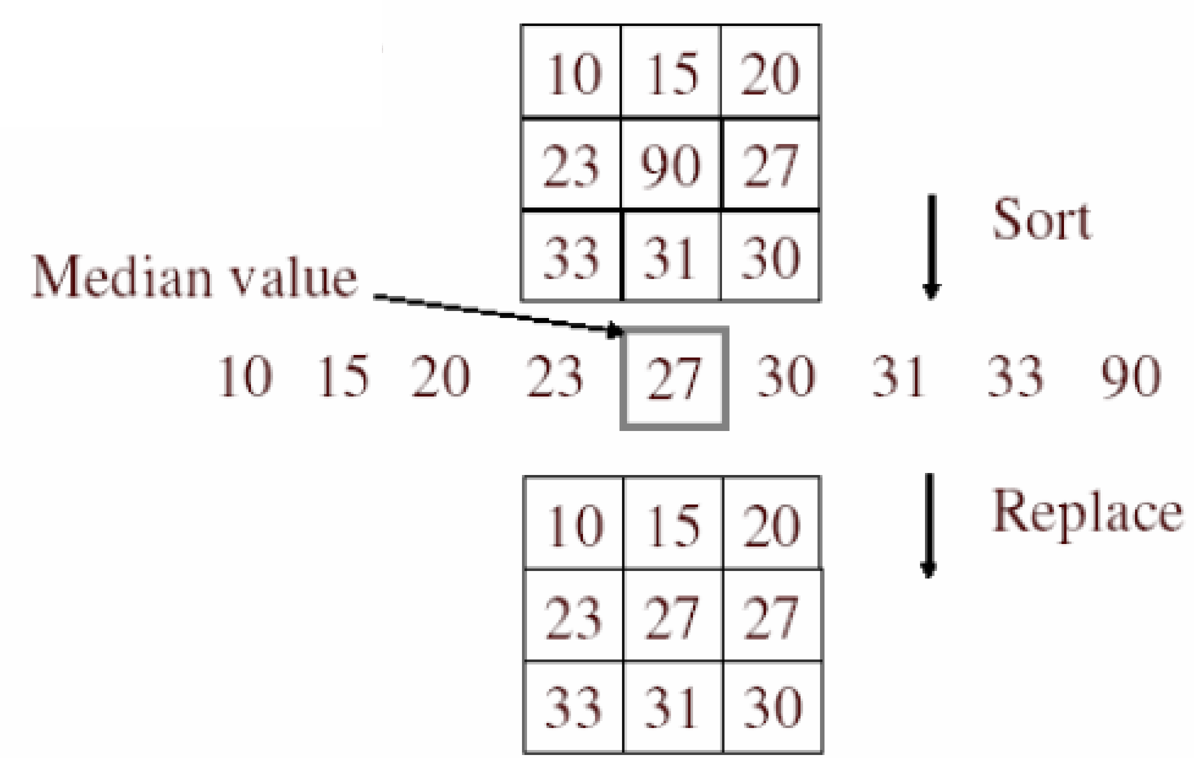
Alternative idea: Median filtering

- A **median filter** operates over a window by selecting the median intensity in the window



- Is median filtering linear?

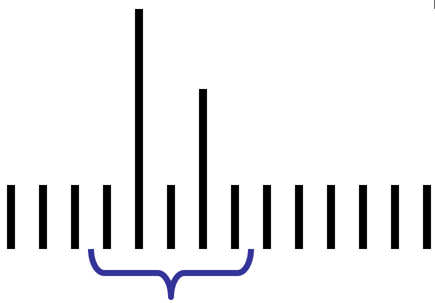
Source: K. Grauman

Median filter

Replace each pixel by the median over N pixels (5 pixels, for these examples). Generalizes to "rank order" filters.

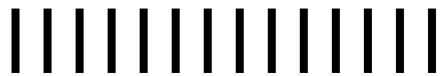
Median([1 7 1 5 1]) = 1
Mean([1 7 1 5 1]) = 2.8

In:



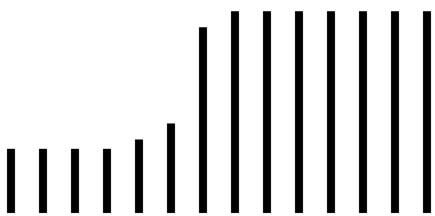
5-pixel neighborhood

Out:

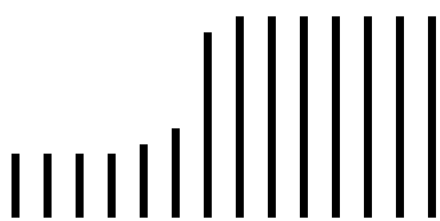


Spike noise is removed

In:



Out:



Monotonic edges remain unchanged

Median filtering results

Best for salt and pepper noise

http://homepages.inf.ed.ac.uk/rbf/HIPR2/mean.htm#guidelines

Median vs. Gaussian filtering

3x3 5x5 7x7

Gaussian

Median

Edges

http://todayinart.com/files/2009/12/500x388xblind-contour-line-drawing.png.pagespeed.ic.DOIi66Ckz1.png

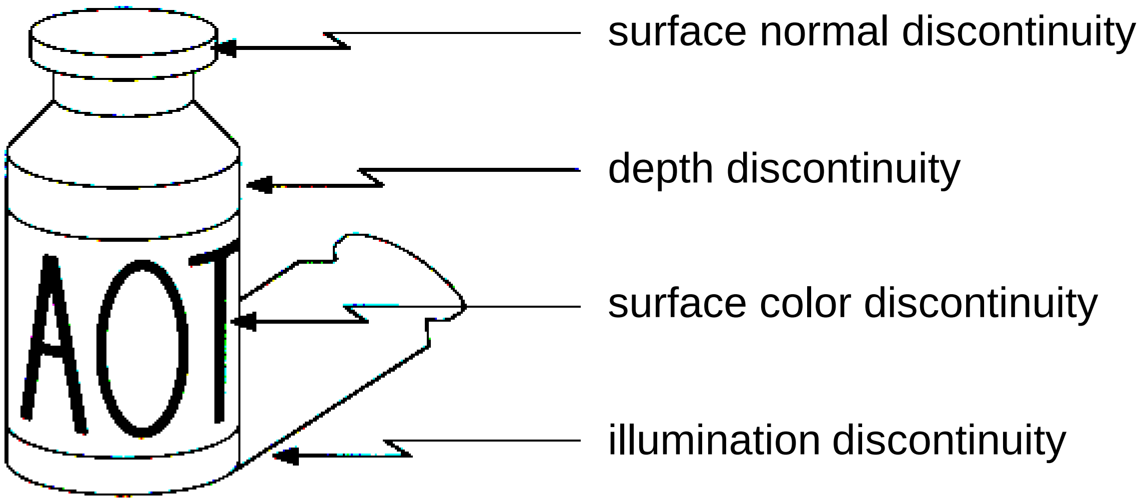
Edge detection

- **Goal:** Identify sudden changes (discontinuities) in an image
  - Intuitively, most semantic and shape information from the image can be encoded in the edges
  - More compact than pixels
- **Ideal:** artist's line drawing (but artist is also using object-level knowledge)

Source: D. Lowe

Origin of edges

Edges are caused by a variety of factors:

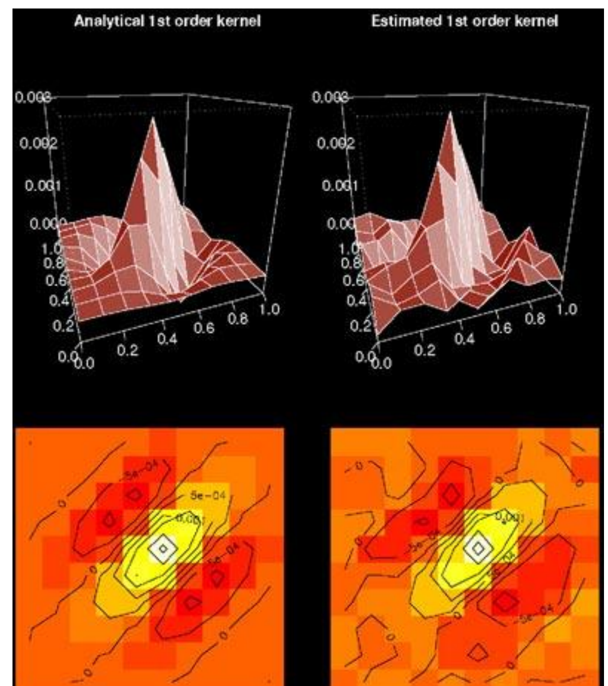


Source: Steve Seitz

Edges in the Visual Cortex

Extract compact, generic, representation of image that carries sufficient information for higher-level processing tasks

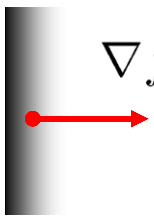
Essentially what area V1 does in our visual cortex.



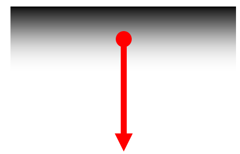
http://www.usc.edu/programs/vpl/private/photos/research/retinal_circuits/figure_2.jpg

Image gradient

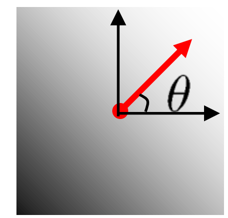
The gradient of an image: $\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$



$$\nabla f = \left[\frac{\partial f}{\partial x}, 0\right]$$



$$\nabla f = \left[0, \frac{\partial f}{\partial y}\right]$$



$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

The gradient points in the direction of most rapid increase in intensity

- How does this direction relate to the direction of the edge?

The gradient direction is given by $\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$

The *edge strength* is given by the gradient magnitude

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

Source: Steve Seitz

Differentiation and convolution

Recall, for 2D function, f(x,y):

$$\frac{\partial f}{\partial x} = \lim_{\varepsilon \to 0}\left(\frac{f(x+\varepsilon, y)}{\varepsilon} - \frac{f(x,y)}{\varepsilon} \right)$$

This is linear and shift invariant, so must be the result of a convolution.

We could approximate this as

$$\frac{\partial f}{\partial x} \approx \frac{f(x_{n+1}, y) - f(x_n, y)}{\Delta x}$$

(which is obviously a convolution)

-1	**1**

Source: D. Forsyth, D. Lowe

Finite difference filters

Other approximations of derivative filters exist:

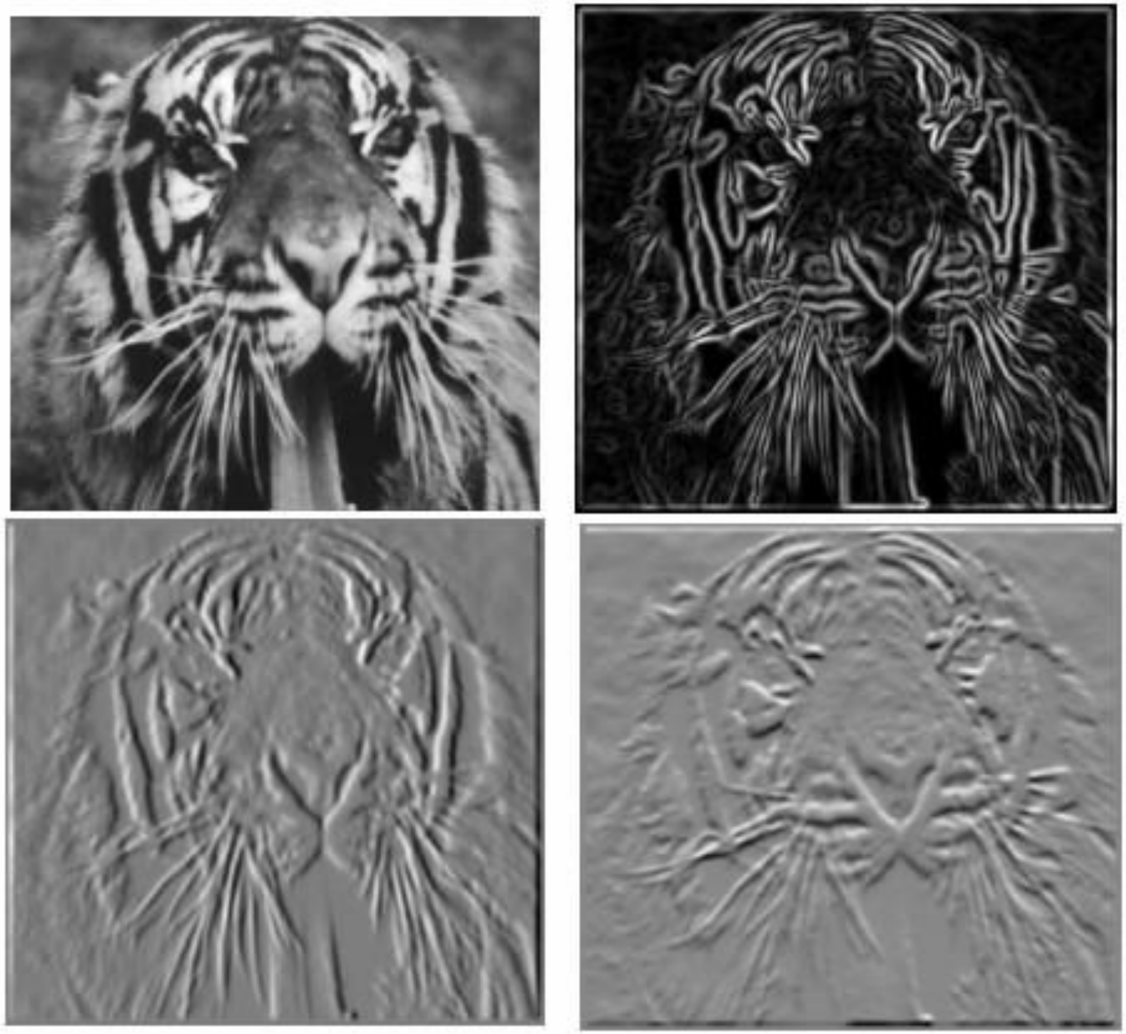
Prewitt: $M_x = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$; $M_y = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ -1 & -1 & -1 \end{bmatrix}$

Sobel: $M_x = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$; $M_y = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$

Roberts: $M_x = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$; $M_y = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

Source: K. Grauman

Finite differences: example



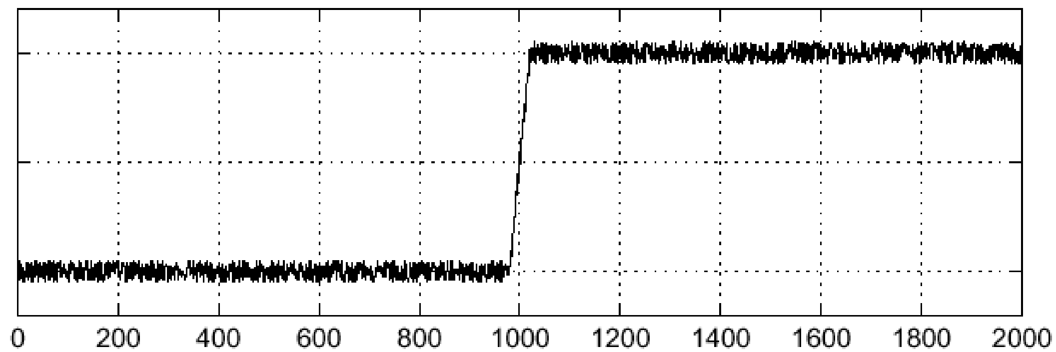
Which one is the gradient in the x-direction (resp. y-direction)?

Effects of noise

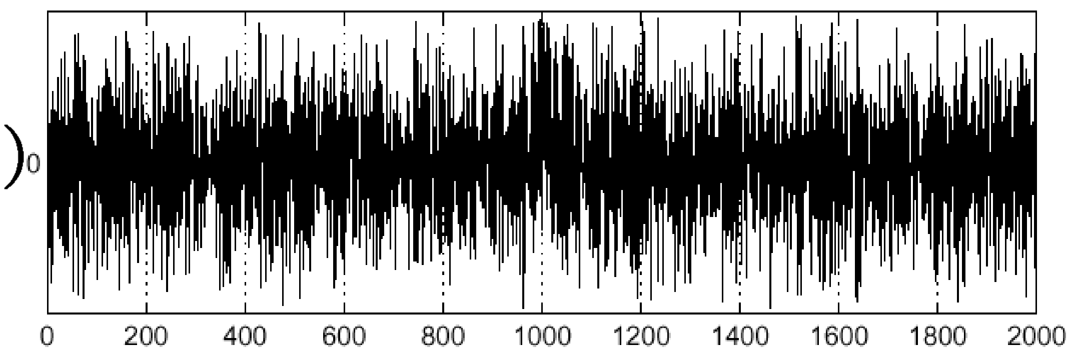
Consider a single row or column of the image

- Plotting intensity as a function of position gives a signal

$f(x)$



$\frac{d}{dx}f(x)$



Where is the edge?

Source: S. Seitz

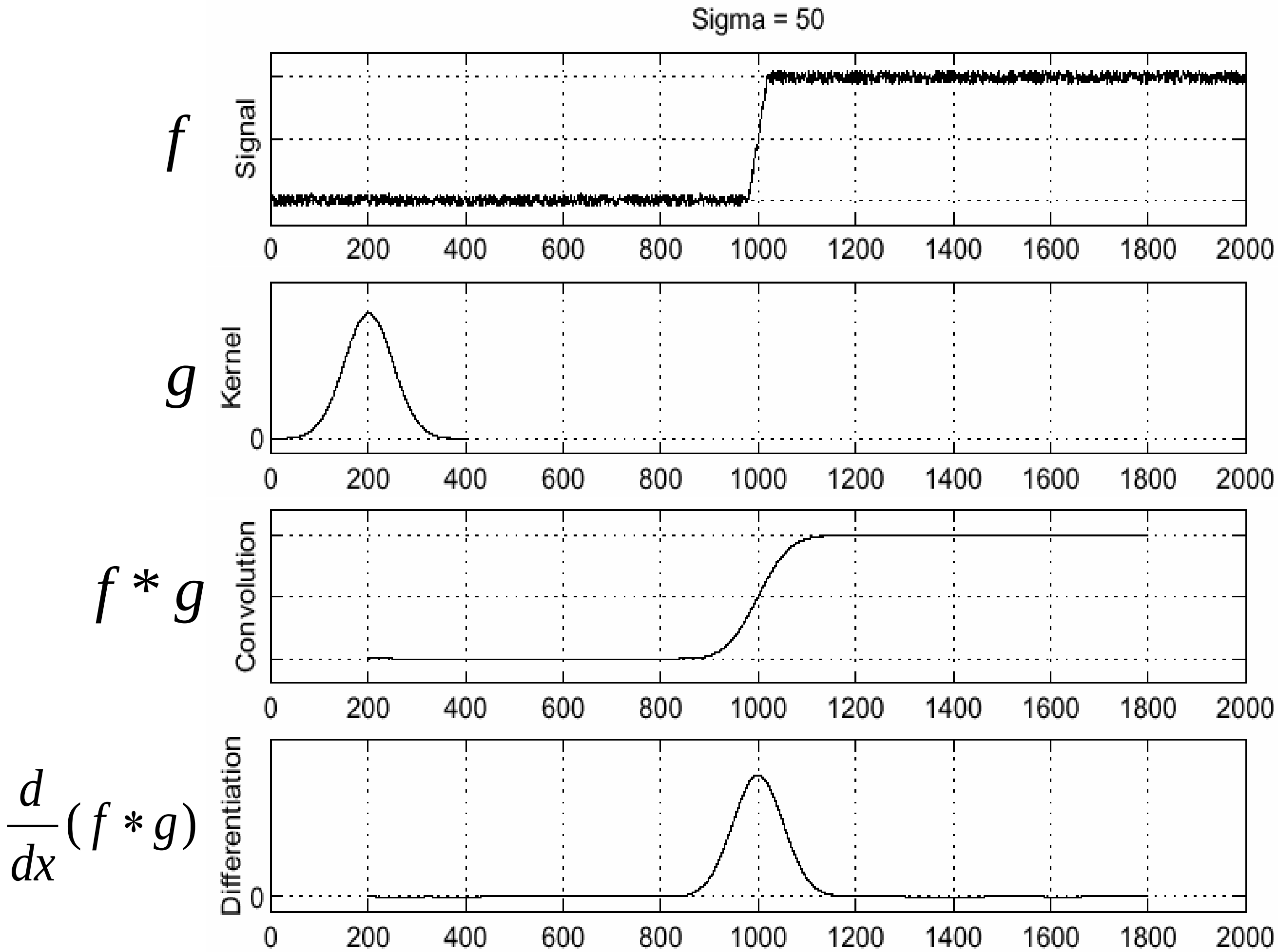
Effects of noise

- Finite difference filters respond strongly to noise
  - Image noise results in pixels that look very different from their neighbors
  - Generally, the larger the noise the stronger the response

- What is to be done?
  - Smoothing the image should help, by forcing pixels different from their neighbors (=noise pixels?) to look more like neighbors

Source: D. Forsyth

Solution: smooth first



- To find edges, look for peaks in $\frac{d}{dx}(f * g)$

Source: S. Seitz

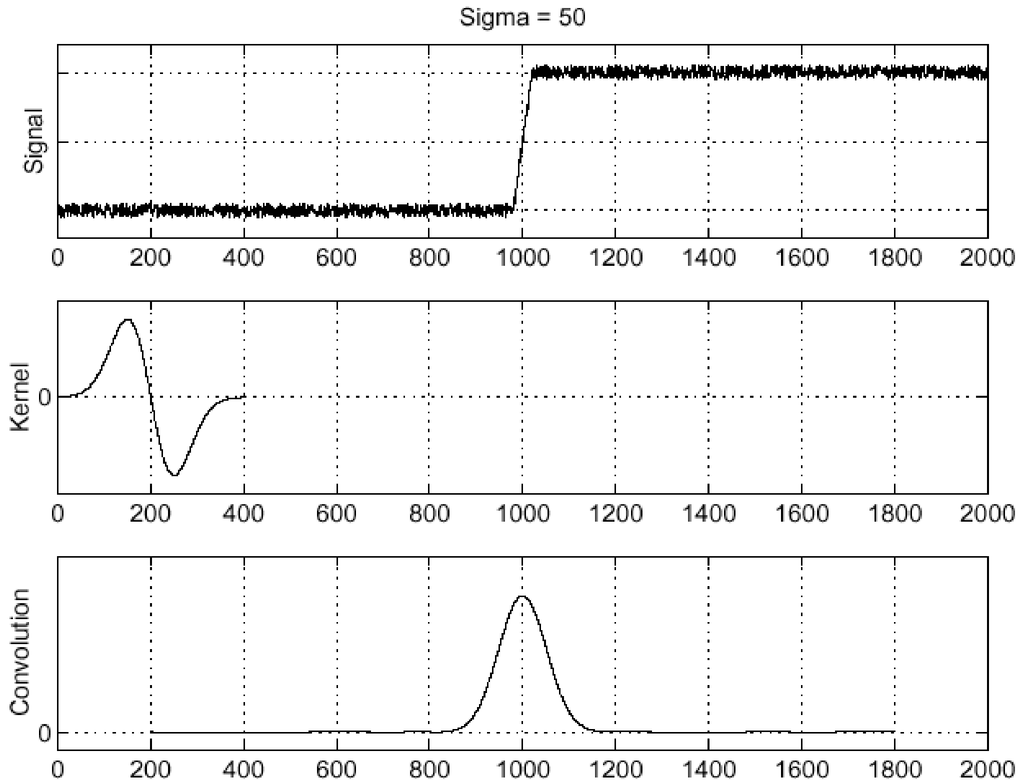
Derivative theorem of convolution

- Differentiation is convolution, and convolution is associative: $$\frac{d}{dx}(f * g) = f * \frac{d}{dx} g$$
- This saves us one operation:

f

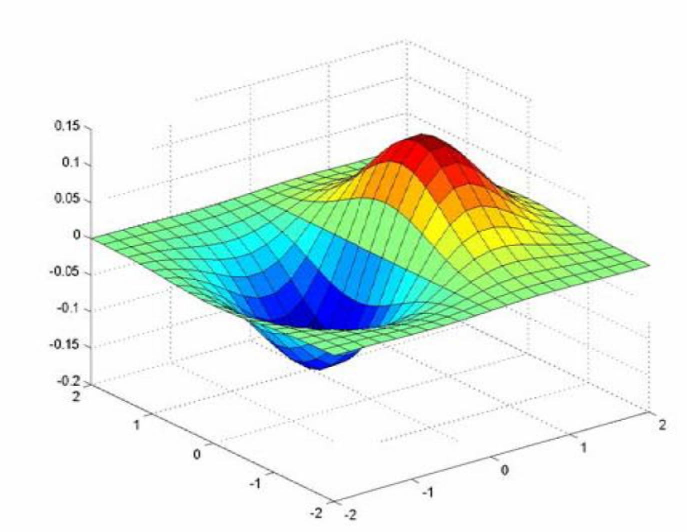
$\frac{d}{dx} g$

$f * \frac{d}{dx} g$

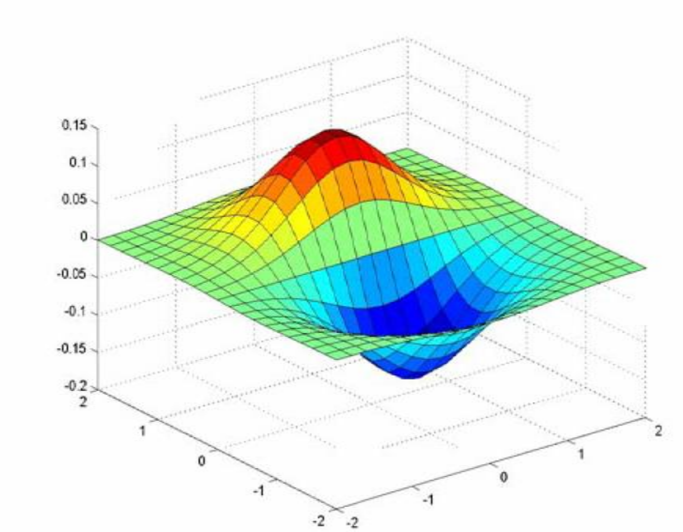


Source: S. Seitz

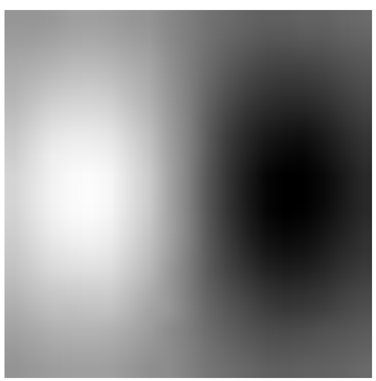
Derivative of Gaussian filter

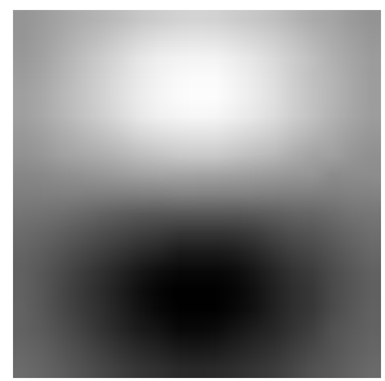


x-direction



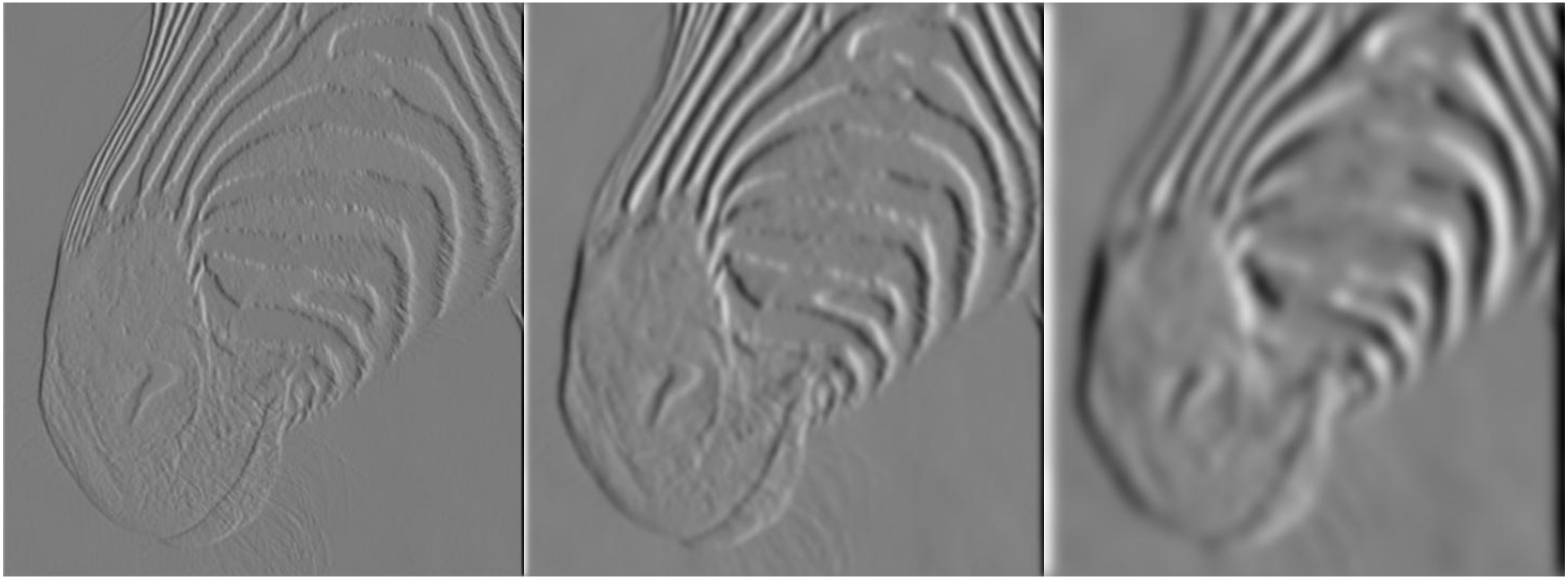
y-direction





Which one finds horizontal/vertical edges?

Scale of Gaussian derivative filter



1 pixel 3 pixels 7 pixels

Smoothed derivative removes noise, but blurs edge. Also finds edges at different “scales”.

Source: D. Forsyth

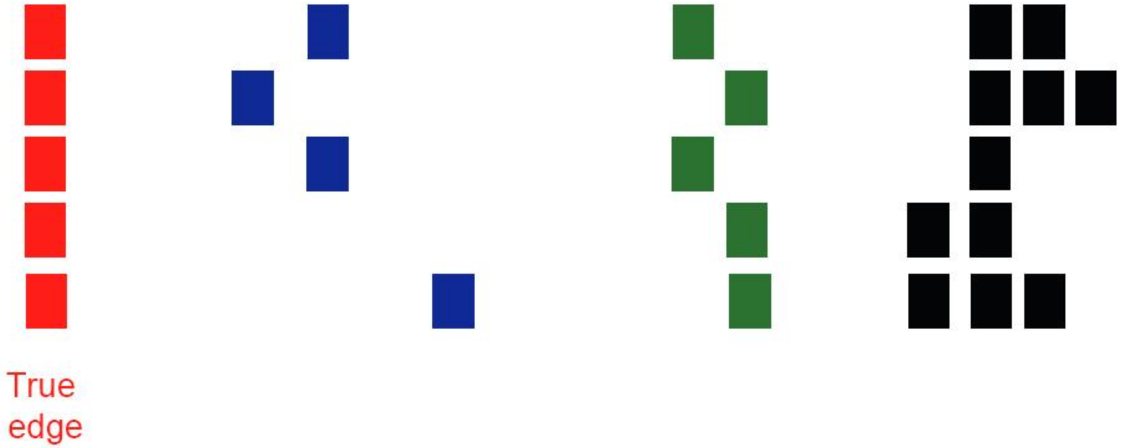
Implementation issues

- The gradient magnitude is large along a thick “trail” or “ridge,” so how do we identify the actual edge points?
- How do we link the edge points to form curves?

Source: D. Forsyth

Designing an edge detector

- Criteria for an "optimal" edge detector:
  - **Good detection:** the optimal detector must minimize the probability of false positives (detecting spurious edges caused by noise), as well as that of false negatives (missing real edges)
  - **Good localization:** the edges detected must be as close as possible to the true edges
  - **Single response:** the detector must return one point only for each true edge point; that is, minimize the number of local maxima around the true edge



Source: L. Fei-Fei

Canny edge detector

- This is probably the most widely used edge detector in computer vision
- Theoretical model: step-edges corrupted by additive Gaussian noise
- Canny has shown that the first derivative of the Gaussian closely approximates the operator that optimizes the product of *signal-to-noise ratio* and localization
- MATLAB: edge(image, ‘canny’)

J. Canny, ***A Computational Approach To Edge Detection***, IEEE Trans. Pattern Analysis and Machine Intelligence, 8:679-714, 1986.

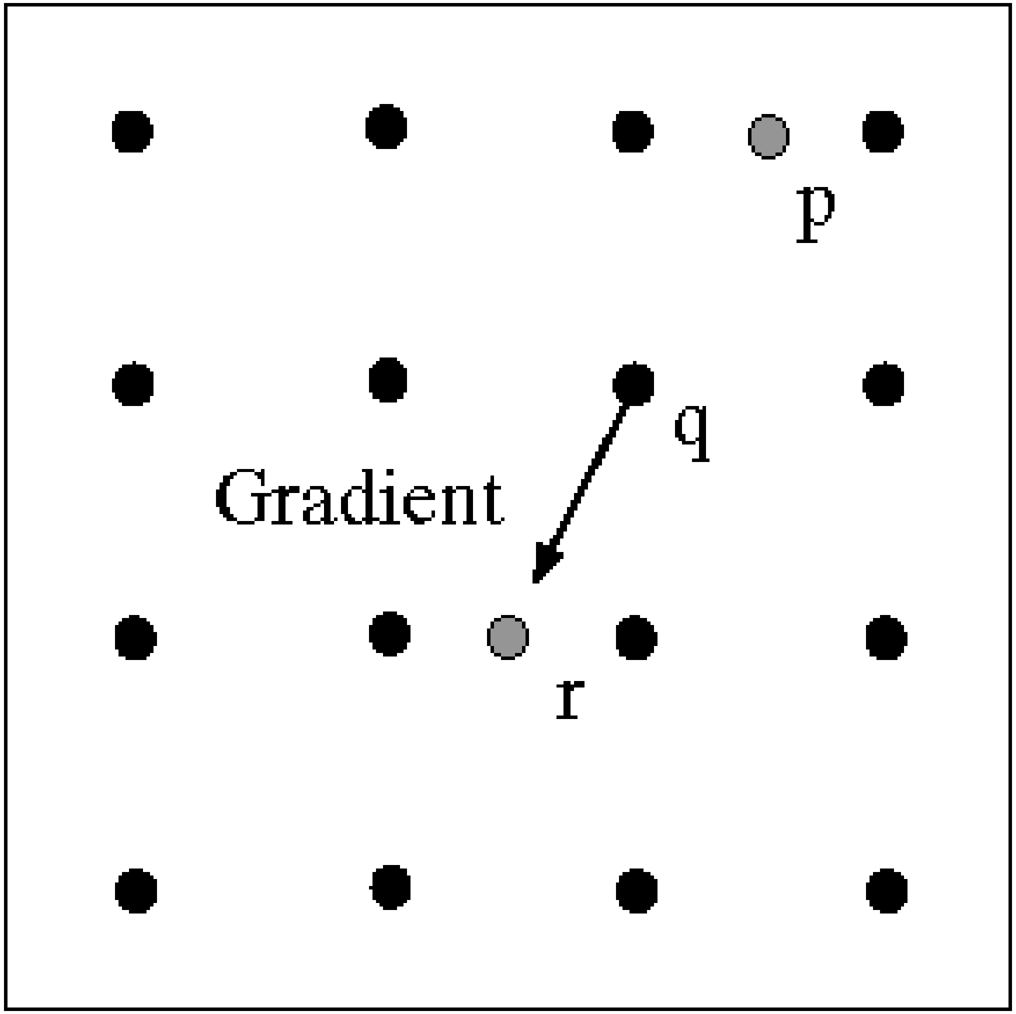
Source: L. Fei-Fei

Canny edge detector

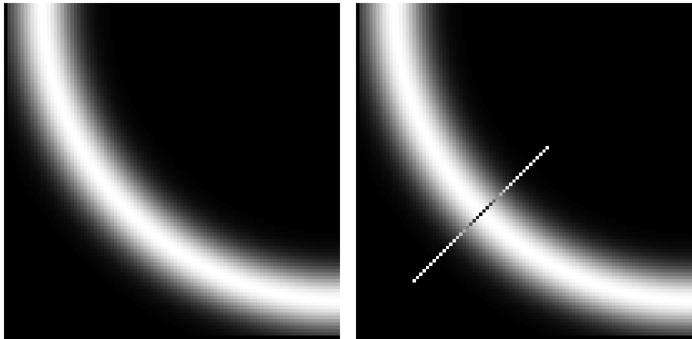
1. Filter image with derivative of Gaussian
2. Find magnitude and orientation of gradient
3. Non-maximum suppression:
   - Thin multi-pixel wide “ridges” down to single pixel width

Source: D. Lowe, L. Fei-Fei

Non-maximum suppression



At q, we have a maximum if the value is larger than those at both p and at r. Interpolate to get these values.



Source: D. Forsyth

Example

original image (Lena)

Example

norm of the gradient

Example

thresholding

Example

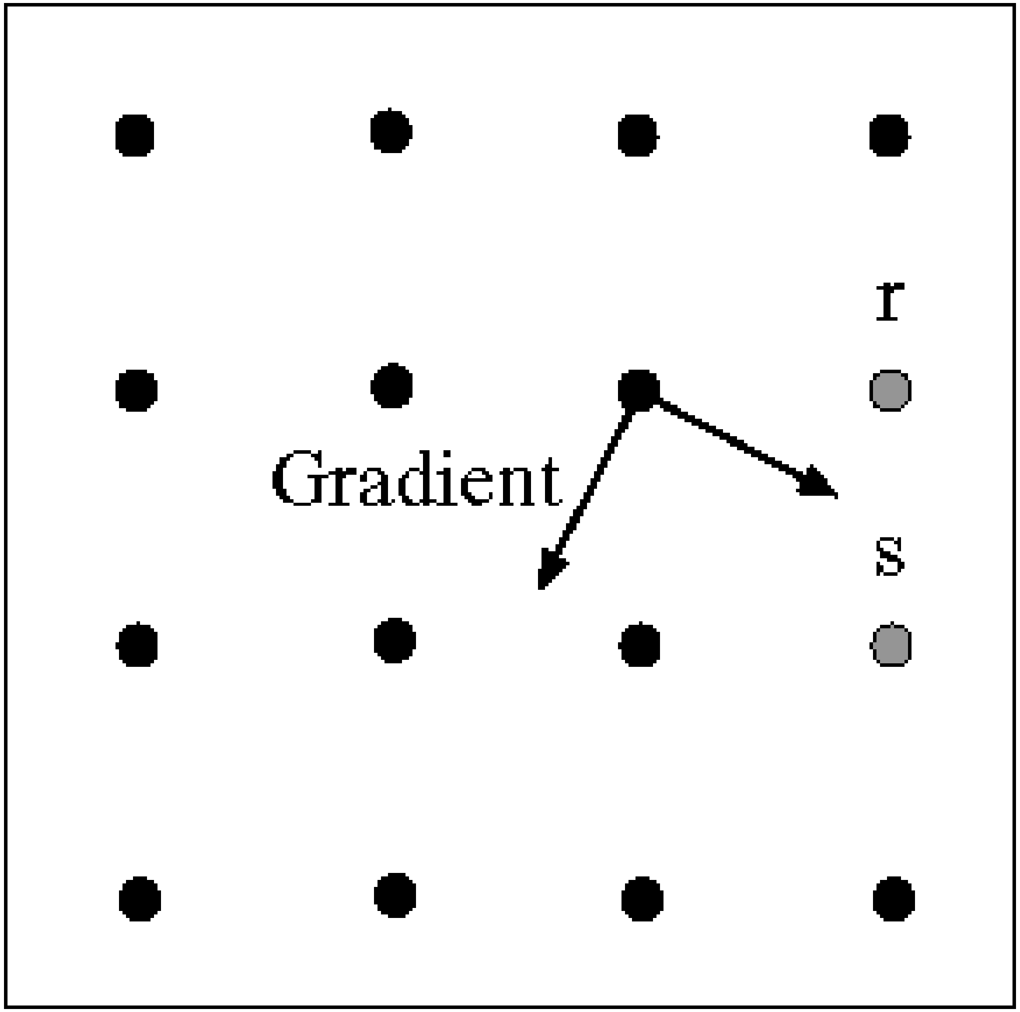
Non-maximum suppression

Canny edge detector

1. Filter image with derivative of Gaussian
2. Find magnitude and orientation of gradient
3. Non-maximum suppression
    - Thin multi-pixel wide “ridges” down to single pixel width
4. Linking of edge points

Source: D. Lowe, L. Fei-Fei

Edge linking



Assume the marked point is an edge point. Then we construct the tangent to the edge curve (which is normal to the gradient at that point) and use this to predict the next points (here either r or s).

Source: D. Forsyth

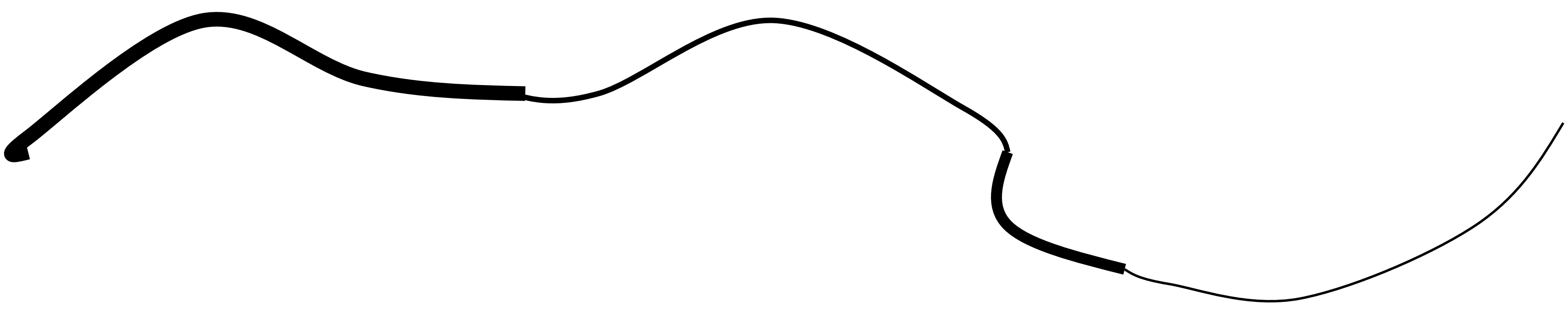
Canny edge detector

1. Filter image with derivative of Gaussian
2. Find magnitude and orientation of gradient
3. Non-maximum suppression
   - Thin multi-pixel wide “ridges” down to single pixel width
4. Linking of edge points
   - Hysteresis thresholding: use a higher threshold to start edge curves and a lower threshold to continue them

Source: D. Lowe, L. Fei-Fei

Hysteresis thresholding

- Use a high threshold to start edge curves and a low threshold to continue them
  - Reduces *drop-outs*



Source: S. Seitz

Hysteresis thresholding

original image

high threshold
(strong edges)

low threshold
(weak edges)

hysteresis threshold

Source: L. Fei-Fei

Effect of σ (Gaussian kernel spread/size)

original | Canny with $\sigma = 1$ | Canny with $\sigma = 2$

The choice of σ depends on desired behavior

- large σ detects large scale edges
- small σ detects fine features

Source: S. Seitz

Edge detection is just the beginning…

image | human segmentation | gradient magnitude

Berkeley segmentation database:

http://www.eecs.berkeley.edu/Research/Projects/CS/vision/grouping/segbench/