Lecture 13 – Optical Flow

With slides from R. Szeliski,
S. Lazebnik, S. Seitz, A. Efros,
C. Liu & F. Durand

Admin

- Assignment 3 due

- Assignment 4 out
  - Deadline: Thursday 11th Dec
  - THIS IS A HARD DEADLINE
    (I have to hand in grades on 12th)

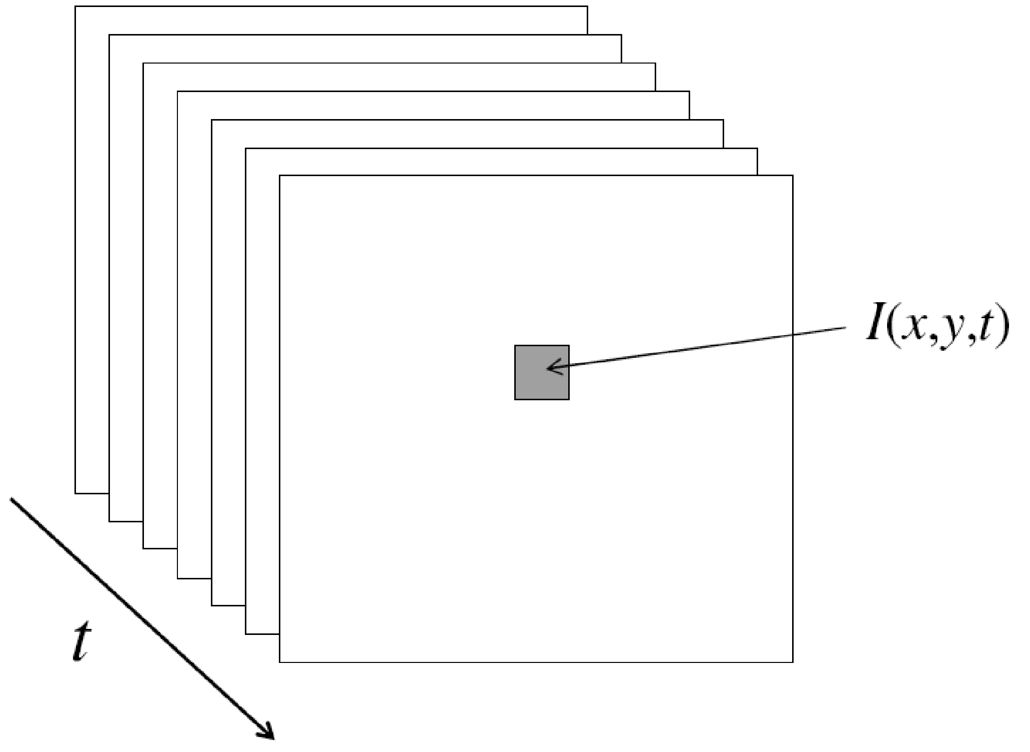
- Course assessment forms

Overview

- Segmentation in Video
- Optical flow
- Motion Magnification

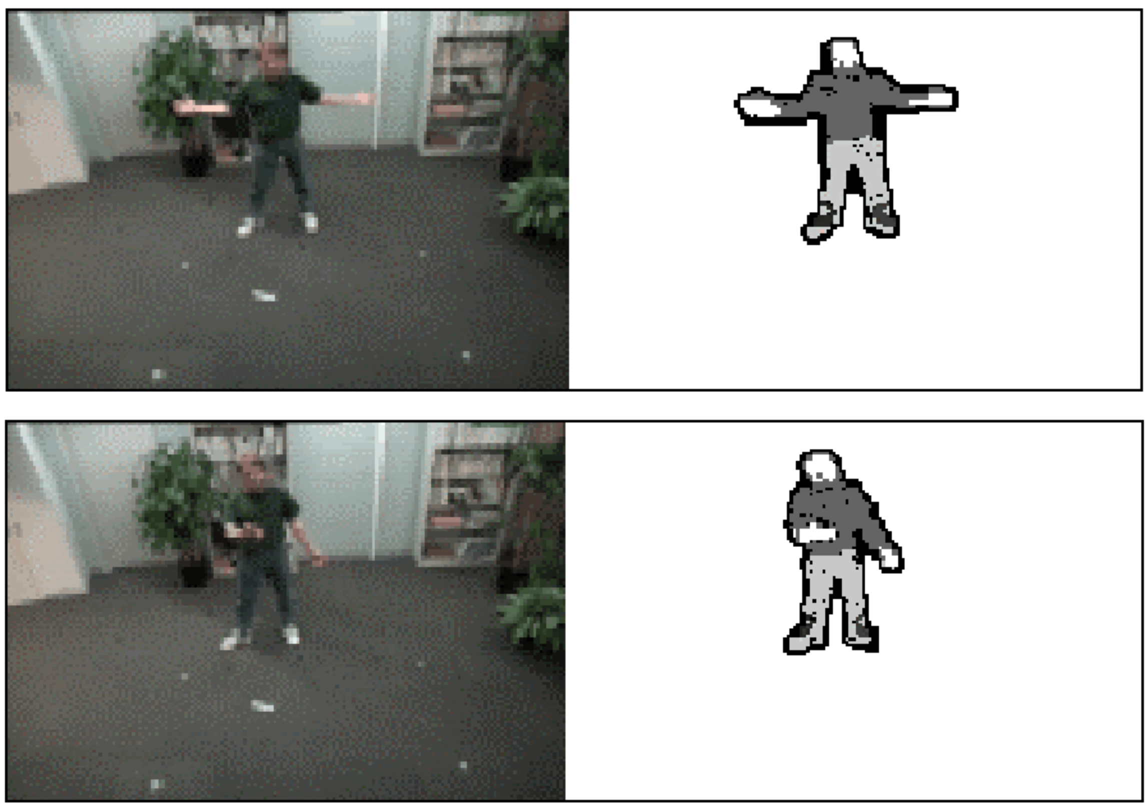
Video

- A video is a sequence of frames captured over time
- Now our image data is a function of space (x, y) and time (t)



Applications of segmentation to video

- Background subtraction
  - A static camera is observing a scene
  - Goal: separate the static *background* from the moving *foreground*

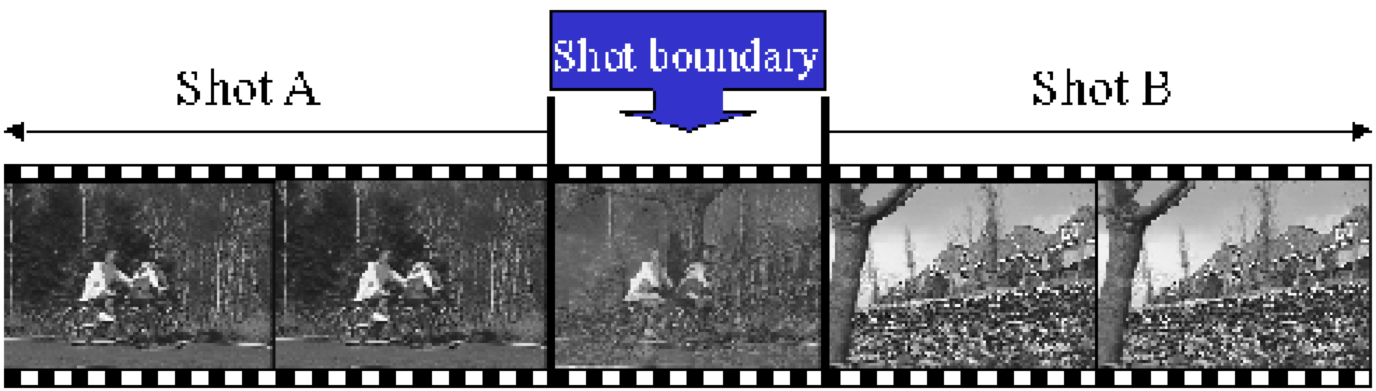


Applications of segmentation to video

- Background subtraction
  - Form an initial background estimate
  - For each frame:
    - Update estimate using a moving average
    - Subtract the background estimate from the frame
    - Label as foreground each pixel where the magnitude of the difference is greater than some threshold
    - Use median filtering to “clean up” the results

Applications of segmentation to video

- Background subtraction
- Shot boundary detection
  - Commercial video is usually composed of *shots* or sequences showing the same objects or scene
  - Goal: segment video into shots for summarization and browsing (each shot can be represented by a single keyframe in a user interface)
  - Difference from background subtraction: the camera is not necessarily stationary

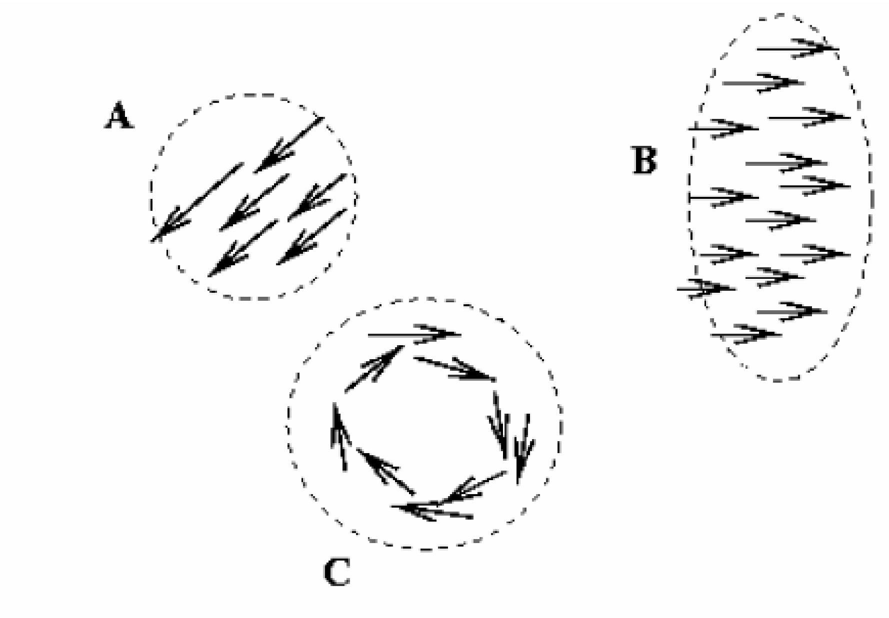


Applications of segmentation to video

- Background subtraction
- Shot boundary detection
  - For each frame
    - Compute the distance between the current frame and the previous one
      - Pixel-by-pixel differences
      - Differences of color histograms
      - Block comparison
    - If the distance is greater than some threshold, classify the frame as a shot boundary

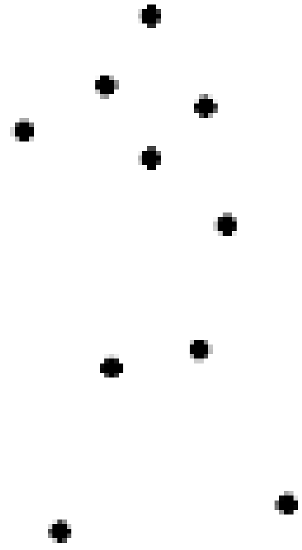
Applications of segmentation to video

- Background subtraction
- Shot boundary detection
- Motion segmentation
  - Segment the video into multiple *coherently* moving objects



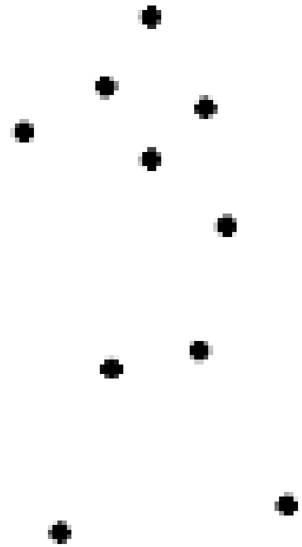
Motion and perceptual organization

- Even “impoverished” motion data can evoke a strong percept



Motion and perceptual organization

- Even “impoverished” motion data can evoke a strong percept



Uses of motion

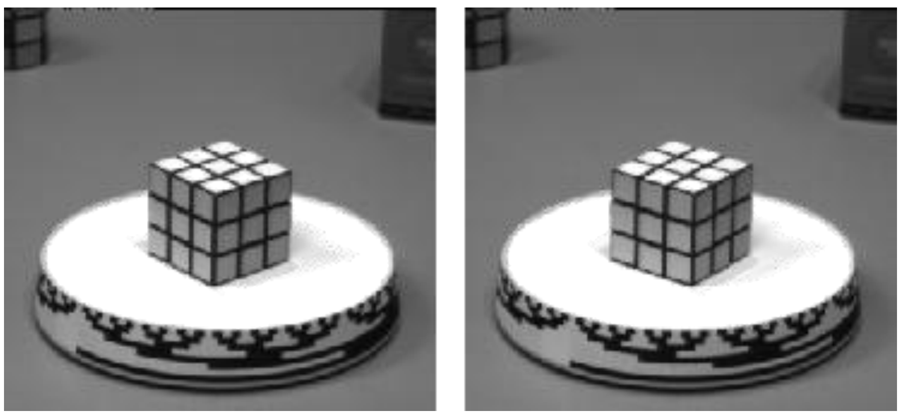
- Estimating 3D structure
- Segmenting objects based on motion cues
- Learning dynamical models
- Recognizing events and activities
- Improving video quality (motion stabilization)

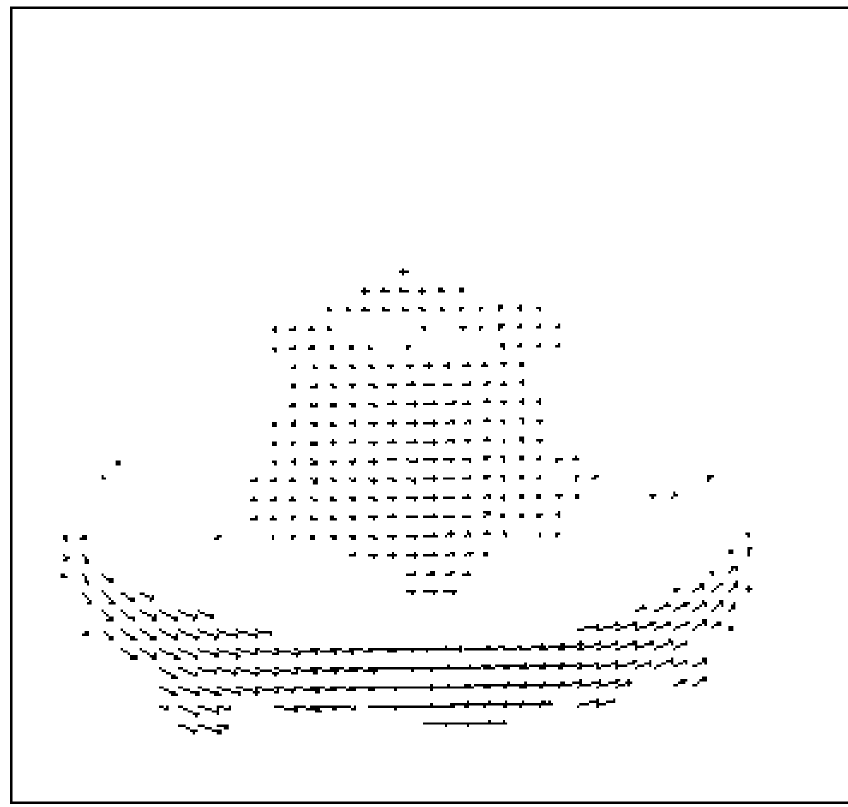
Motion estimation techniques

- Direct methods
  - Directly recover image motion at each pixel from spatio-temporal image brightness variations
  - Dense motion fields, but sensitive to appearance variations
  - Suitable for video and when image motion is small
- Feature-based methods
  - Extract visual features (corners, textured areas) and track them over multiple frames
  - Sparse motion fields, but more robust tracking
  - Suitable when image motion is large (10s of pixels)

Motion field

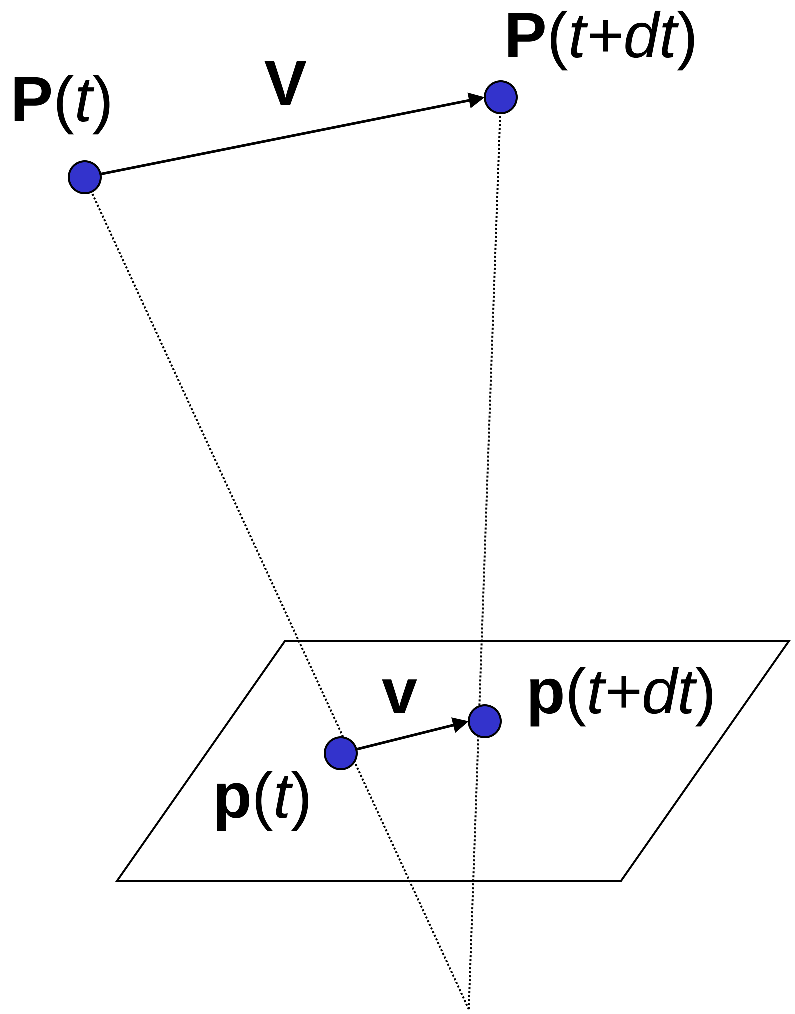
- The motion field is the projection of the 3D scene motion into the image





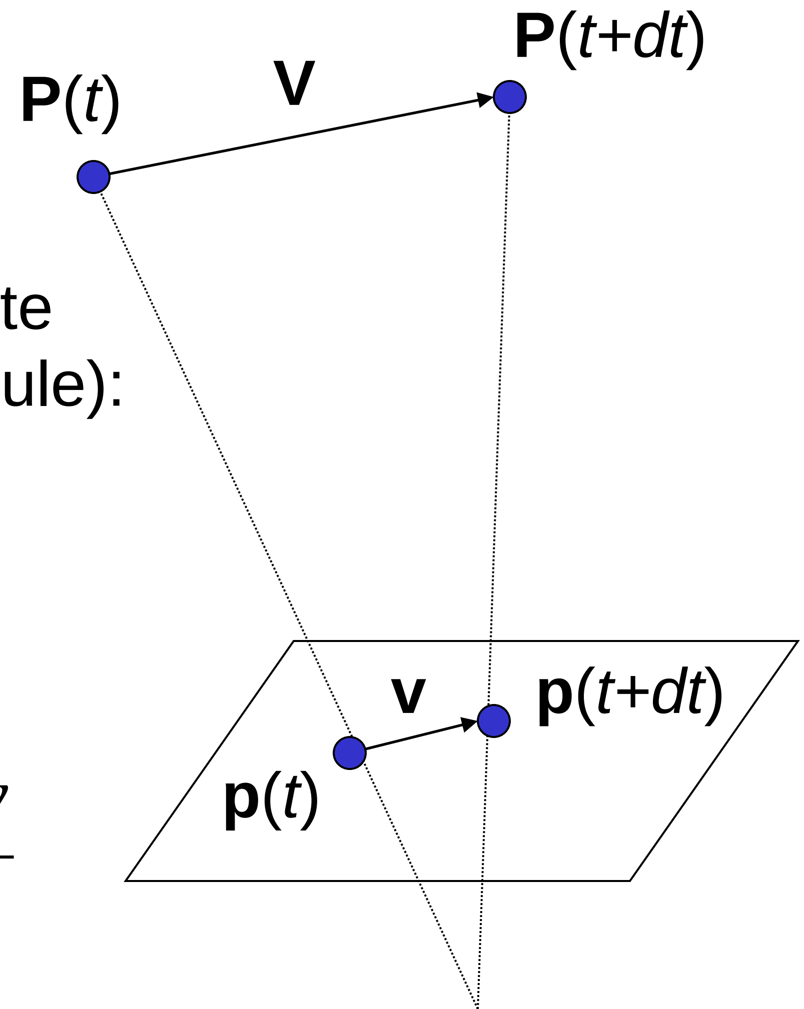
Motion field and parallax

- $\mathbf{P}(t)$ is a moving 3D point
- Velocity of scene point: $\mathbf{V} = \mathrm{d}\mathbf{P}/\mathrm{d}t$
- $\mathbf{p}(t) = (x(t),y(t))$ is the projection of $\mathbf{P}$ in the image
- Apparent velocity $\mathbf{v}$ in the image: given by components $v_x = \mathrm{d}x/\mathrm{d}t$ and $v_y = \mathrm{d}y/\mathrm{d}t$
- These components are known as the *motion field* of the image



Motion field and parallax

$$\mathbf{V} = (V_x, V_y, V_Z) \qquad \mathbf{p} = f\,\frac{\mathbf{P}}{Z}$$



To find image velocity **v**, differentiate **p** with respect to *t* (using quotient rule):

$$\mathbf{v} = f\,\frac{Z\mathbf{V} - V_z\mathbf{P}}{Z^2}$$

$$v_x = \frac{fV_x - V_z x}{Z} \qquad v_y = \frac{fV_y - V_z y}{Z}$$

Image motion is a function of both the 3D motion (**V**) and the depth of the 3D point (*Z*)

Motion field and parallax

- Pure translation: **V** is constant everywhere

$$v_x = \frac{fV_x - V_z x}{Z}$$

$$v_y = \frac{fV_y - V_z y}{Z}$$

$$\mathbf{v} = \frac{1}{Z}(\mathbf{v}_0 - V_z \mathbf{p}),$$

$$\mathbf{v}_0 = (fV_x, fV_y)$$

Motion field and parallax

- Pure translation: **V** is constant everywhere

$$\mathbf{v} = \frac{1}{Z}(\mathbf{v}_0 - V_z \mathbf{p}),$$

$$\mathbf{v}_0 = (f V_x, f V_y)$$

- V_z is nonzero:
  - Every motion vector points toward (or away from) $\mathbf{v}_0$, the vanishing point of the translation direction

Motion field and parallax

- Pure translation: **V** is constant everywhere

$$\mathbf{v} = \frac{1}{Z}(\mathbf{v}_0 - V_z\mathbf{p}),$$

$$\mathbf{v}_0 = (fV_x, fV_y)$$

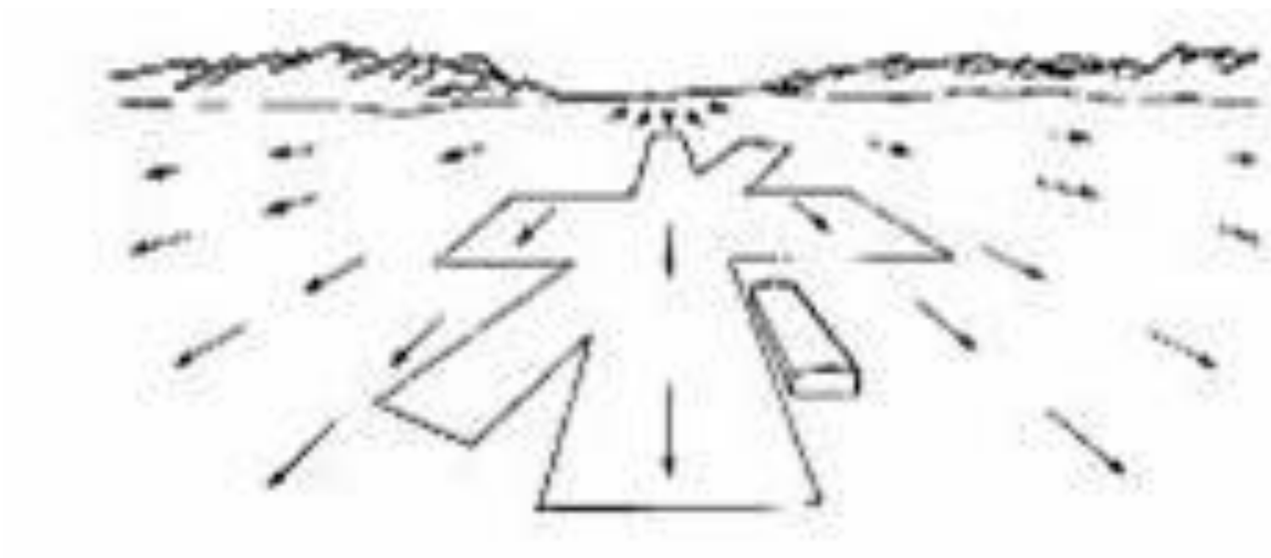
- V_z is nonzero:
  - Every motion vector points toward (or away from) $\mathbf{v}_0$, the vanishing point of the translation direction
- V_z is zero:
  - Motion is parallel to the image plane, all the motion vectors are parallel
- The length of the motion vectors is inversely proportional to the depth Z

Overview

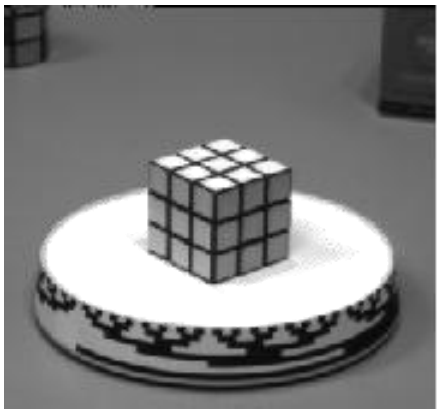
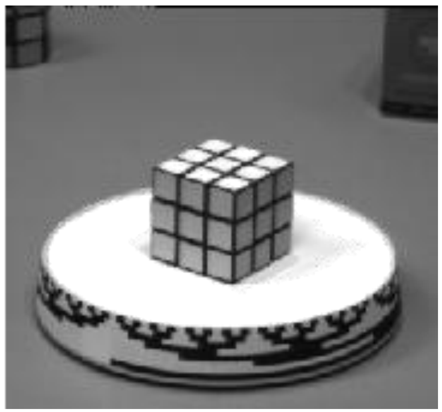
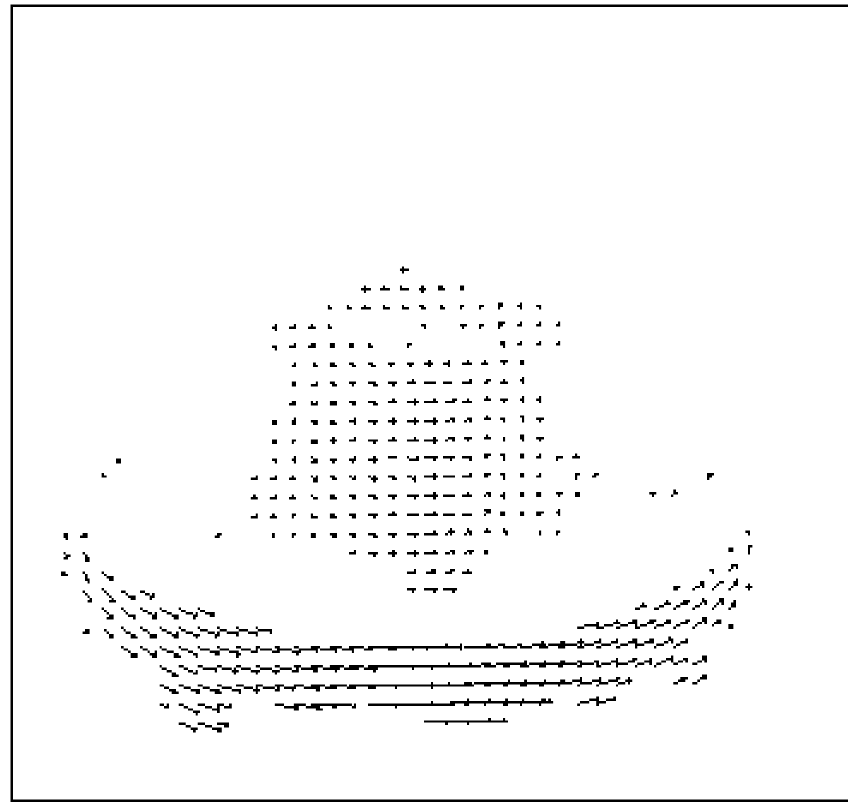
- Segmentation in Video
- Optical flow
- Motion Magnification

Optical flow

Combination of slides from Rick Szeliski, Steve Seitz, Alyosha Efros and Bill Freeman and Fredo Durand



Motion estimation: Optical flow

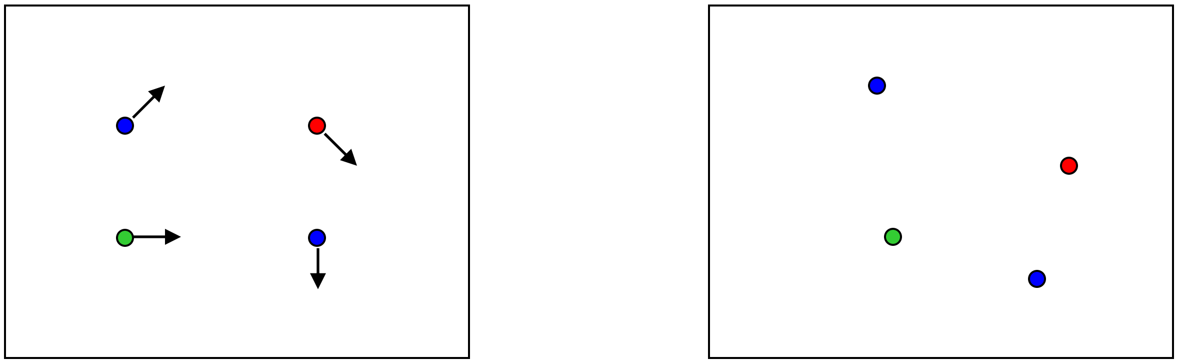
Will start by estimating motion of each pixel separately
Then will consider motion of entire image

Why estimate motion?

Lots of uses

- Track object behavior
- Correct for camera jitter (stabilization)
- Align images (mosaics)
- 3D shape reconstruction
- Special effects

Problem definition: optical flow



$H(x, y)$ $I(x, y)$

How to estimate pixel motion from image H to image I?

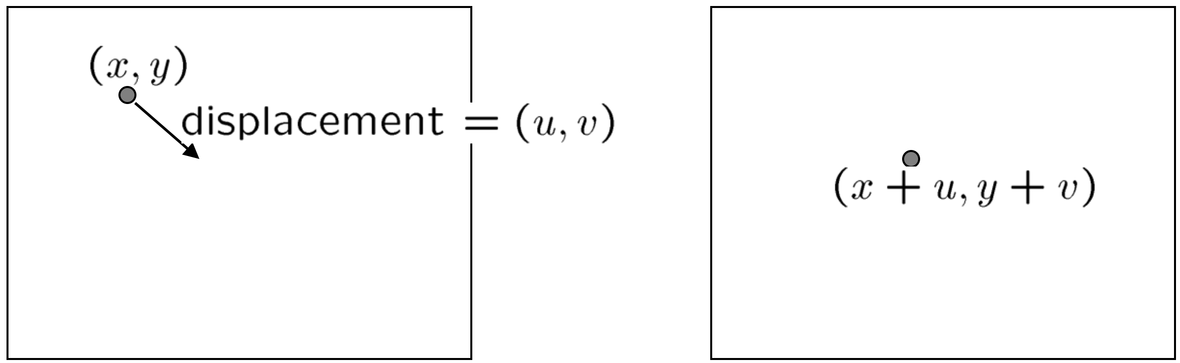
- Solve pixel correspondence problem
  - given a pixel in H, look for nearby pixels of the same color in I

Key assumptions

- color constancy: a point in H looks the same in I
  - For grayscale images, this is brightness constancy
- small motion: points do not move very far

This is called the optical flow problem

Optical flow constraints (grayscale images)



$H(x, y)$ $\qquad\qquad$ $I(x, y)$

Let's look at these constraints more closely

- brightness constancy: Q: what's the equation?

  H(x,y)=I(x+u, y+v)

- small motion: (u and v are less than 1 pixel)
  - suppose we take the Taylor series expansion of I:

$$I(x+u, y+v) = I(x, y) + \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v + \text{higher order terms}$$

$$\approx I(x, y) + \frac{\partial I}{\partial x}u + \frac{\partial I}{\partial y}v$$

Optical flow equation

Combining these two equations

shorthand: $I_x = \frac{\partial I}{\partial x}$

$$0 = I(x+u, y+v) - H(x,y)$$

$$\approx I(x,y) + I_x u + I_y v - H(x,y)$$

$$\approx (I(x,y) - H(x,y)) + I_x u + I_y v$$

$$\approx I_t + I_x u + I_y v$$

$$\approx I_t + \nabla I \cdot [u \;\; v]$$

In the limit as u and v go to zero, this becomes exact

$$0 = I_t + \nabla I \cdot [\tfrac{\partial x}{\partial t} \;\; \tfrac{\partial y}{\partial t}]$$

Optical flow equation

$$0 = I_t + \nabla I \cdot [u \;\; v]$$

Q: how many unknowns and equations per pixel?

2 unknowns, one equation

Intuitively, what does this constraint mean?

- The component of the flow in the gradient direction is determined
- The component of the flow parallel to an edge is unknown

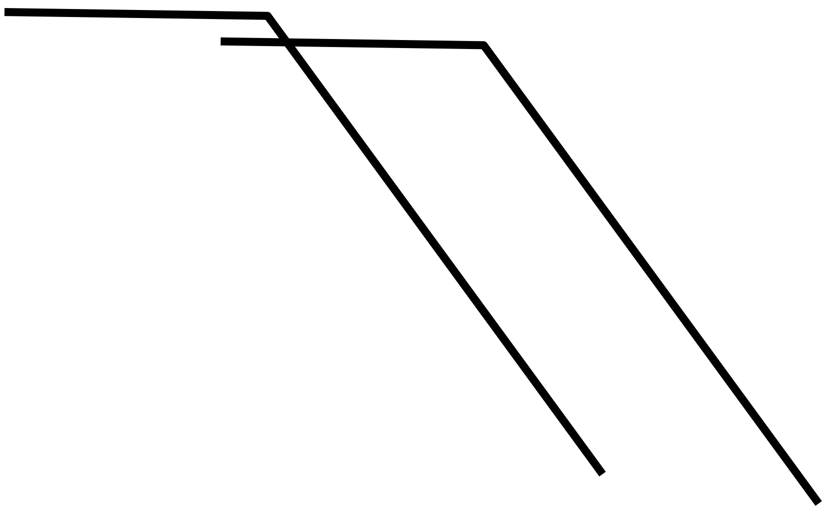
This explains the Barber Pole illusion

http://www.sandlotscience.com/Ambiguous/Barberpole_Illusion.htm

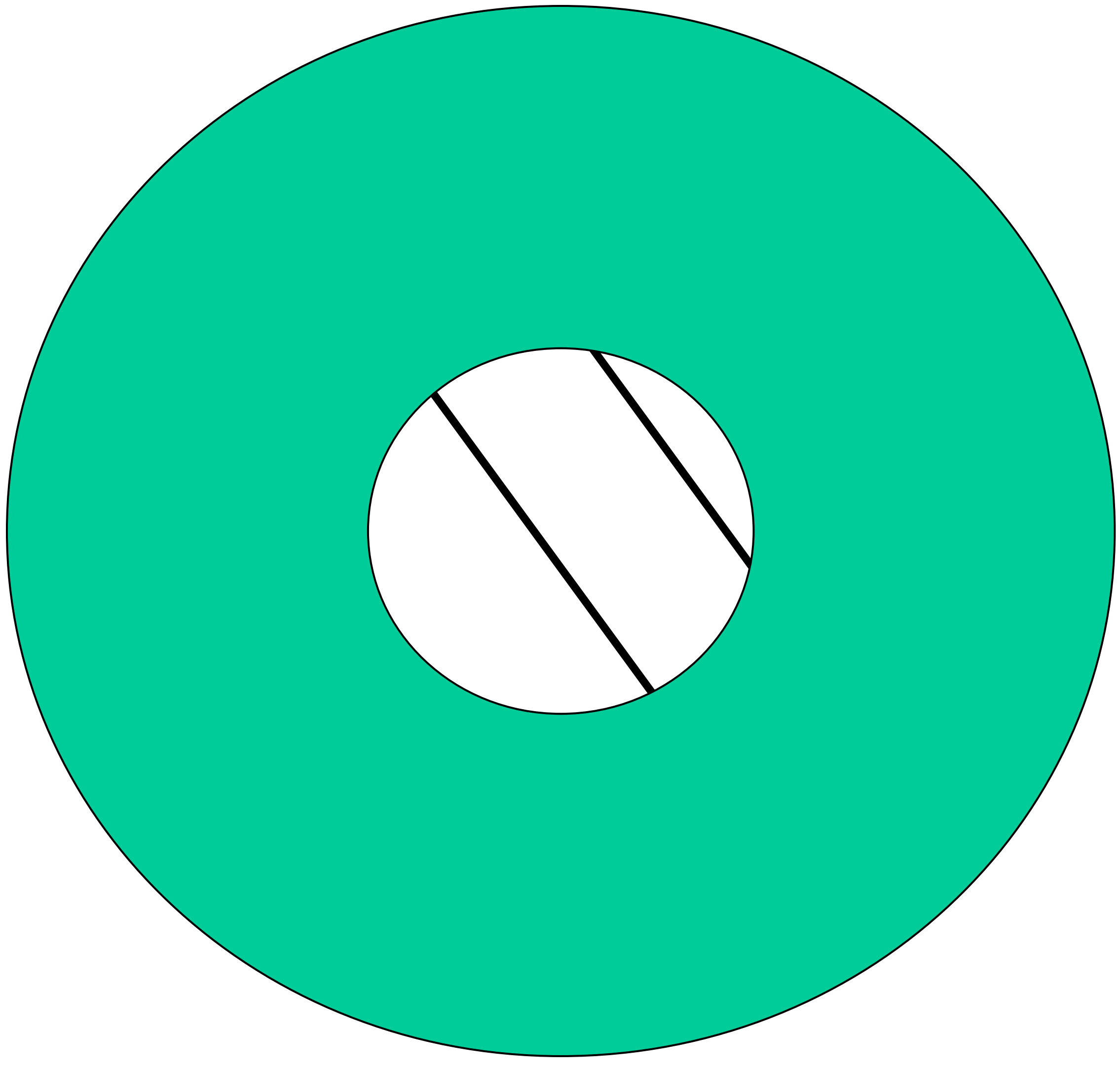
http://www.liv.ac.uk/~marcob/Trieste/barberpole.html

http://en.wikipedia.org/wiki/Barber's_pol

Aperture problem



Aperture problem



Solving the aperture problem

How to get more equations for a pixel?

- Basic idea: impose additional constraints
  - most common is to assume that the flow field is smooth locally
  - one method: pretend the pixel's neighbors have the same (u,v)
    - » If we use a 5x5 window, that gives us 25 equations per pixel!

$$0 = I_t(\mathbf{p_i}) + \nabla I(\mathbf{p_i}) \cdot [u \;\; v]$$

$$\underbrace{\begin{bmatrix} I_x(\mathbf{p_1}) & I_y(\mathbf{p_1}) \\ I_x(\mathbf{p_2}) & I_y(\mathbf{p_2}) \\ \vdots & \vdots \\ I_x(\mathbf{p_{25}}) & I_y(\mathbf{p_{25}}) \end{bmatrix}}_{\substack{A \\ 25\times 2}} \underbrace{\begin{bmatrix} u \\ v \end{bmatrix}}_{\substack{d \\ 2\times 1}} = - \underbrace{\begin{bmatrix} I_t(\mathbf{p_1}) \\ I_t(\mathbf{p_2}) \\ \vdots \\ I_t(\mathbf{p_{25}}) \end{bmatrix}}_{\substack{b \\ 25\times 1}}$$

RGB version

How to get more equations for a pixel?

- Basic idea: impose additional constraints
  - most common is to assume that the flow field is smooth locally
  - one method: pretend the pixel's neighbors have the same (u,v)
    - If we use a 5x5 window, that gives us 25*3 equations per pixel!

$$0 = I_t(\mathbf{p_i})[0,1,2] + \nabla I(\mathbf{p_i})[0,1,2] \cdot [u \ \ v]$$

$$\begin{bmatrix} I_x(\mathbf{p_1})[0] & I_y(\mathbf{p_1})[0] \\ I_x(\mathbf{p_1})[1] & I_y(\mathbf{p_1})[1] \\ I_x(\mathbf{p_1})[2] & I_y(\mathbf{p_1})[2] \\ \vdots & \vdots \\ I_x(\mathbf{p_{25}})[0] & I_y(\mathbf{p_{25}})[0] \\ I_x(\mathbf{p_{25}})[1] & I_y(\mathbf{p_{25}})[1] \\ I_x(\mathbf{p_{25}})[2] & I_y(\mathbf{p_{25}})[2] \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} I_t(\mathbf{p_1})[0] \\ I_t(\mathbf{p_1})[1] \\ I_t(\mathbf{p_1})[2] \\ \vdots \\ I_t(\mathbf{p_{25}})[0] \\ I_t(\mathbf{p_{25}})[1] \\ I_t(\mathbf{p_{25}})[2] \end{bmatrix}$$

$$\underset{75\times 2}{A} \qquad \underset{2\times 1}{d} \qquad \underset{75\times 1}{b}$$

Note that RGB is not enough to disambiguate because R, G & B are correlated
Just provides better gradient

Lukas-Kanade flow

Prob: we have more equations than unknowns

$$\underset{25\times2}{A}\ \underset{2\times1}{d} = \underset{25\times1}{b} \longrightarrow \text{minimize } \|Ad - b\|^2$$

Solution: solve least squares problem

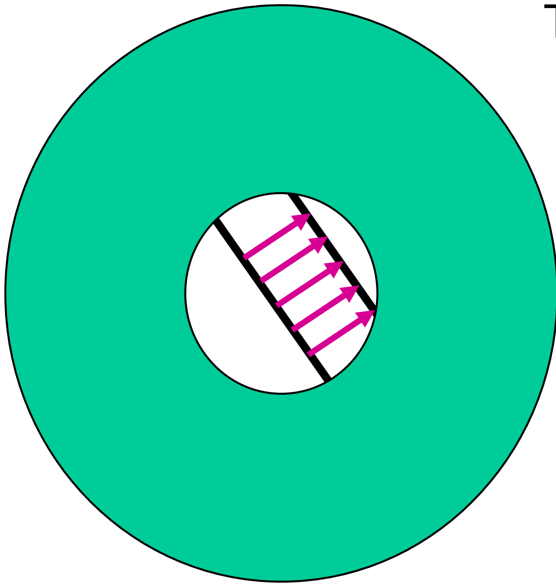
- minimum least squares solution given by solution (in d) of:

$$\underset{2\times2}{(A^T A)}\ \underset{2\times1}{d} = \underset{2\times1}{A^T b}$$

$$\underbrace{\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix}}_{A^T A} \begin{bmatrix} u \\ v \end{bmatrix} = - \underbrace{\begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}}_{A^T b}$$

- The summations are over all pixels in the K x K window
- This technique was first proposed by Lukas & Kanade (1981)

Aperture Problem and Normal Flow



The gradient constraint:

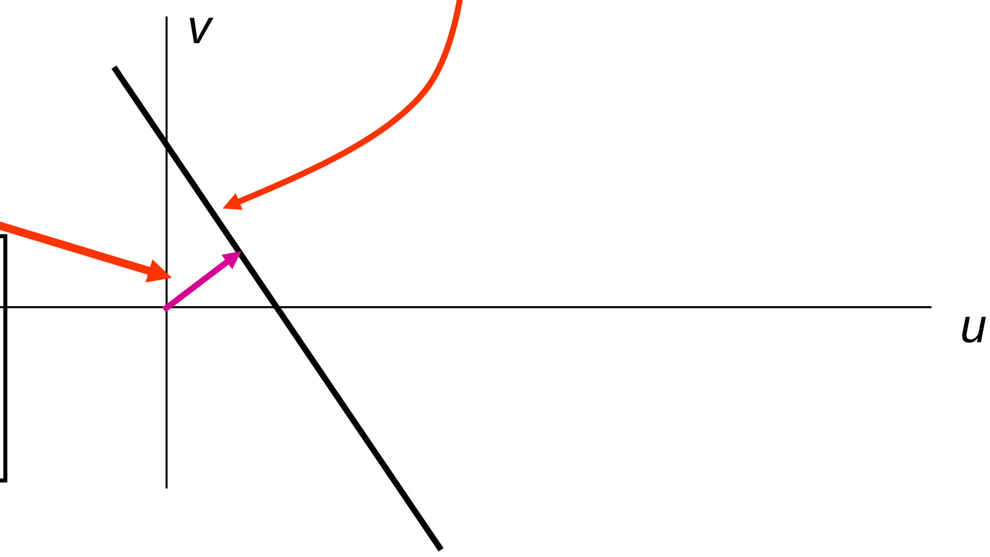
$$I_x u + I_y v + I_t = 0$$

$$\nabla I \bullet \vec{U} = 0$$

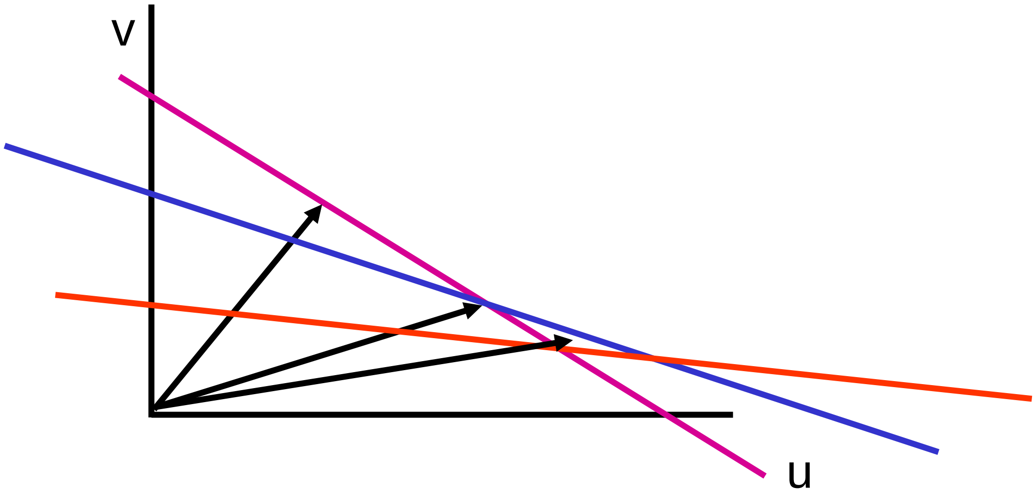
Defines a line in the *(u,v)* space

Normal Flow:

$$u_\perp = -\frac{I_t}{|\nabla I|}\frac{\nabla I}{|\nabla I|}$$



Combining Local Constraints



$$\nabla I^1 \bullet U = -I_t^1$$

$$\nabla I^2 \bullet U = -I_t^2$$

$$\nabla I^3 \bullet U = -I_t^3$$

etc.

Conditions for solvability

- Optimal (u, v) satisfies Lucas-Kanade equation

$$\underbrace{\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix}}_{A^T A} \begin{bmatrix} u \\ v \end{bmatrix} = - \underbrace{\begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}}_{A^T b}$$

When is This Solvable?

- A^TA should be invertible
- A^TA should not be too small due to noise
  – eigenvalues λ_1 and λ_2 of A^TA should not be too small
- A^TA should be well-conditioned
  – λ_1/ λ_2 should not be too large (λ_1 = larger eigenvalue)

A^TA is solvable when there is no aperture problem

$$A^T A = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} = \sum \begin{bmatrix} I_x \\ I_y \end{bmatrix} [I_x \ I_y] = \sum \nabla I (\nabla I)^T$$

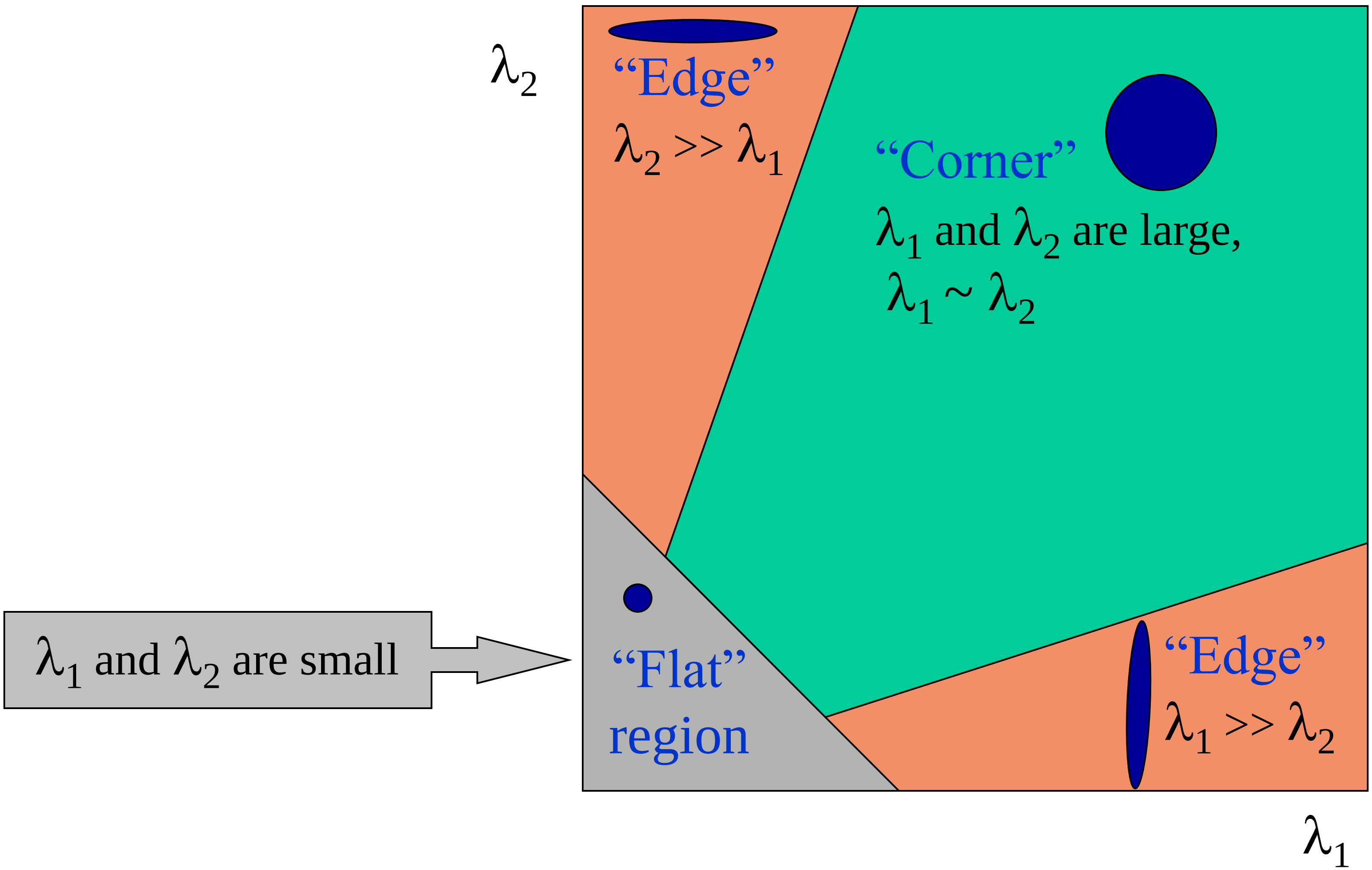
Eigenvectors of A^TA

$$A^TA = \left[\begin{array}{cc} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{array} \right] = \sum \left[\begin{array}{c} I_x \\ I_y \end{array} \right] [I_x \ I_y] = \sum \nabla I (\nabla I)^T$$

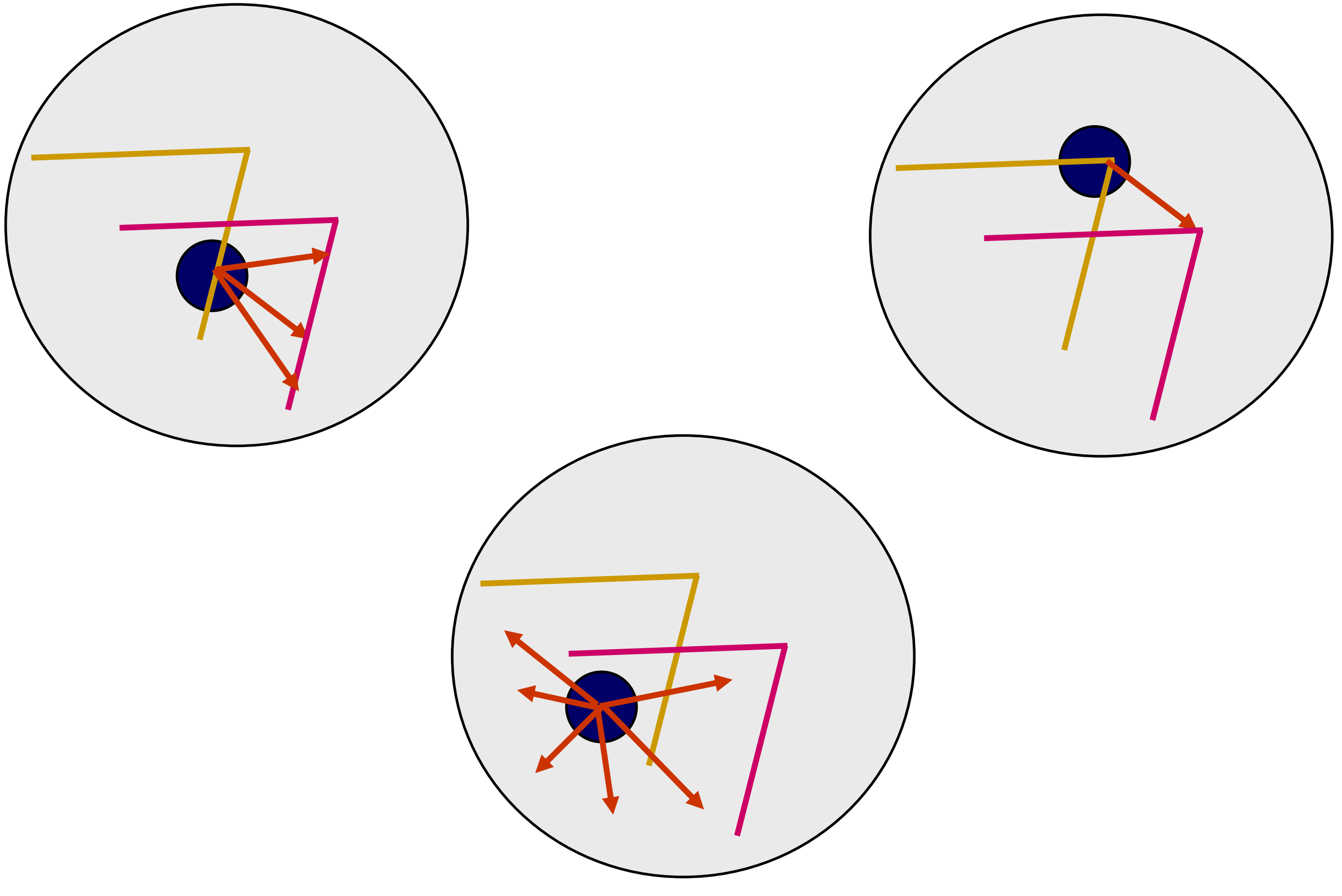
- Recall the Harris corner detector: $M = A^TA$ is the *second moment matrix*
- The eigenvectors and eigenvalues of M relate to edge direction and magnitude
  - The eigenvector associated with the larger eigenvalue points in the direction of fastest intensity change
  - The other eigenvector is orthogonal to it

Interpreting the eigenvalues

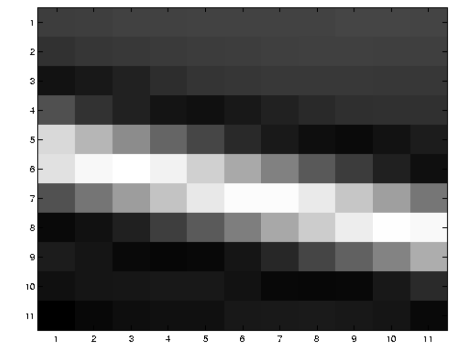
Classification of image points using eigenvalues of the second moment matrix:

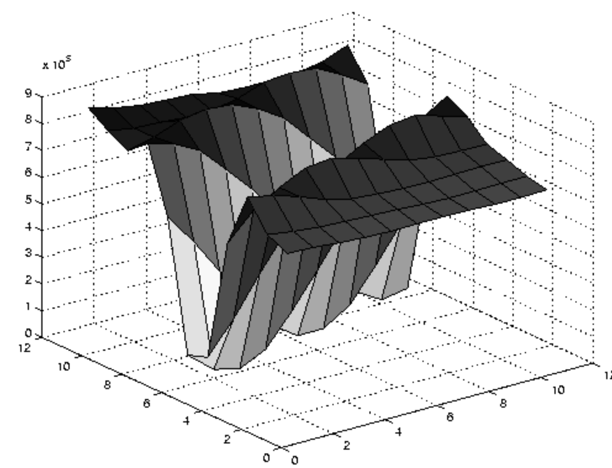


Local Patch Analysis



Edge



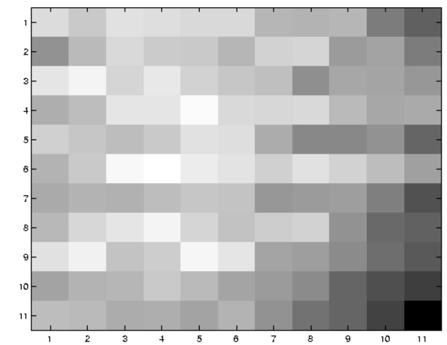


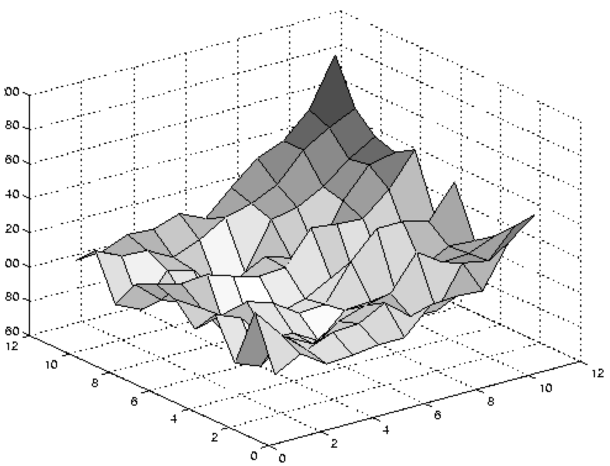
$$\sum \nabla I (\nabla I)^T$$

– large gradients, all the same

– large λ_1, small λ_2

Low texture region



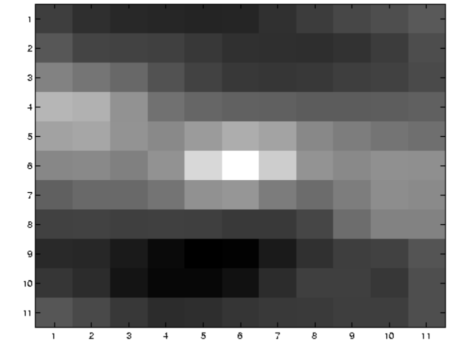


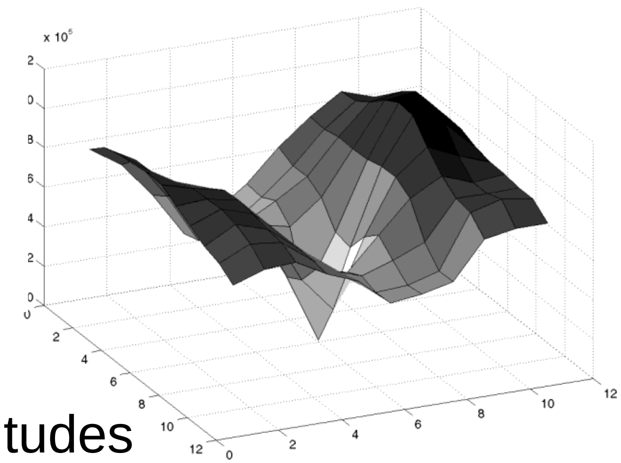
$$\sum \nabla I (\nabla I)^T$$

– gradients have small magnitude

– small λ_1, small λ_2

High textured region





$$\sum \nabla I (\nabla I)^T$$

– gradients are different, large magnitudes

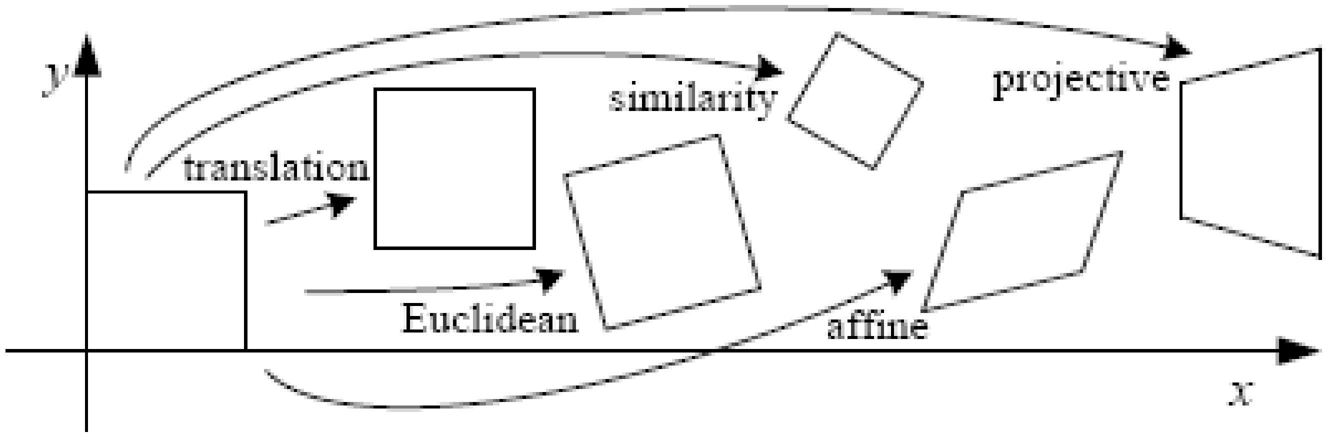
– large λ_1, large λ_2

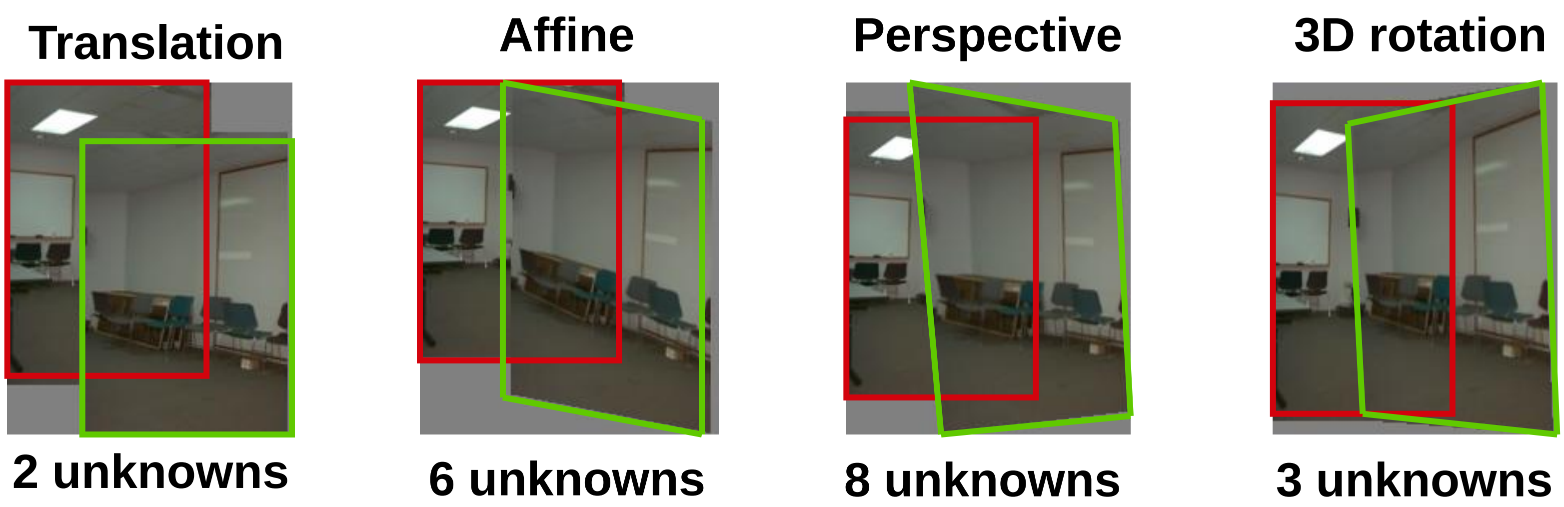
Observation

This is a two image problem BUT

- Can measure sensitivity by just looking at one of the images!
- This tells us which pixels are easy to track, which are hard
  - very useful later on when we do feature tracking...

Motion models

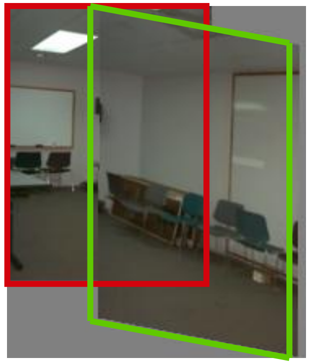




Affine motion

$$u(x, y) = a_1 + a_2 x + a_3 y$$

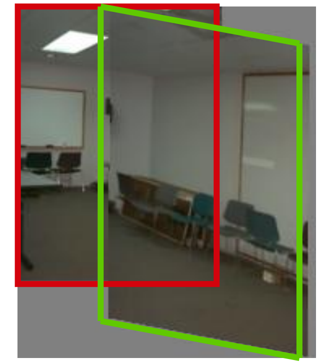
$$v(x, y) = a_4 + a_5 x + a_6 y$$



- Substituting into the brightness constancy equation:

$$\boxed{I_x \cdot u + I_y \cdot v + I_t \approx 0}$$

Affine motion



$$u(x, y) = a_1 + a_2 x + a_3 y$$

$$v(x, y) = a_4 + a_5 x + a_6 y$$

- Substituting into the brightness constancy equation:

$$\boxed{I_x(a_1 + a_2 x + a_3 y) + I_y(a_4 + a_5 x + a_6 y) + I_t \approx 0}$$

- Each pixel provides 1 linear constraint in 6 unknowns

- Least squares minimization:

$$\boxed{Err(\vec{a}) = \sum \left[I_x(a_1 + a_2 x + a_3 y) + I_y(a_4 + a_5 x + a_6 y) + I_t \right]^2}$$

Errors in Lukas-Kanade

What are the potential causes of errors in this procedure?

- Suppose A^TA is easily invertible
- Suppose there is not much noise in the image

When our assumptions are violated

- Brightness constancy is not satisfied
- The motion is not small
- A point does not move like its neighbors
  - window size is too large
  - what is the ideal window size?

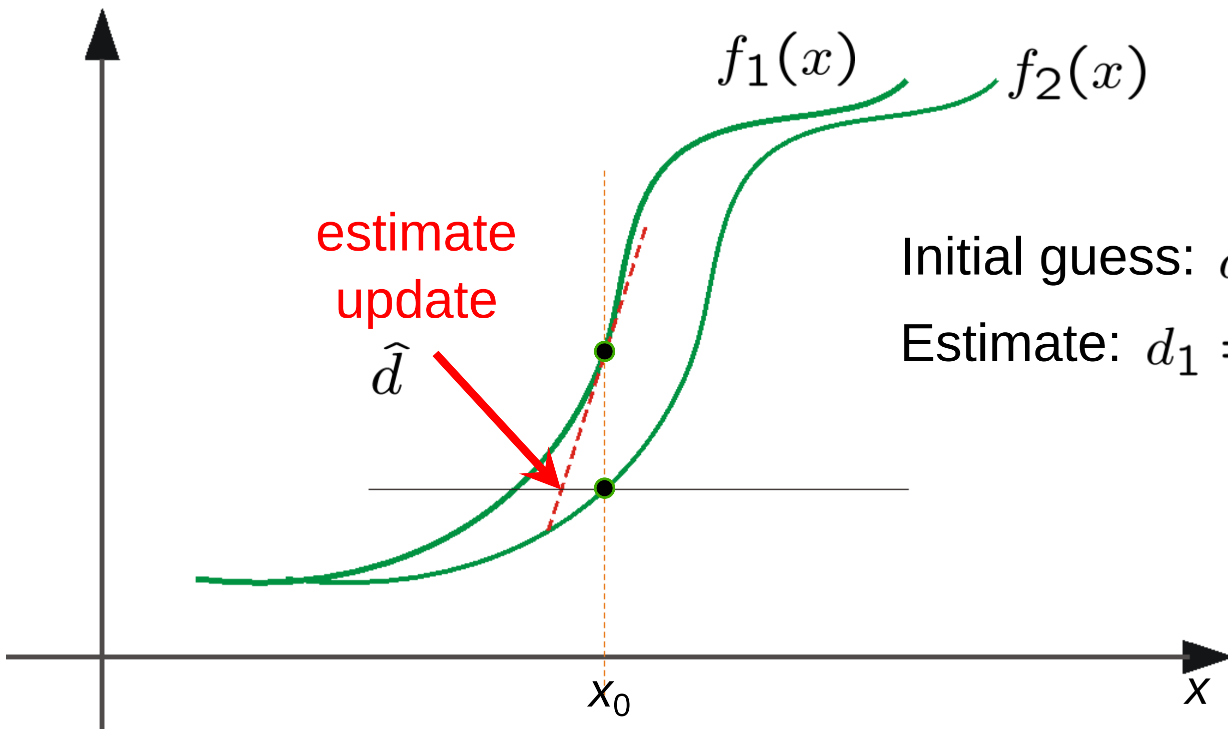
Iterative Refinement

Iterative Lukas-Kanade Algorithm

1. Estimate velocity at each pixel by solving Lucas-Kanade equations
2. Warp H towards I using the estimated flow field

   - use image warping techniques
3. Repeat until convergence

Optical Flow: Iterative Estimation

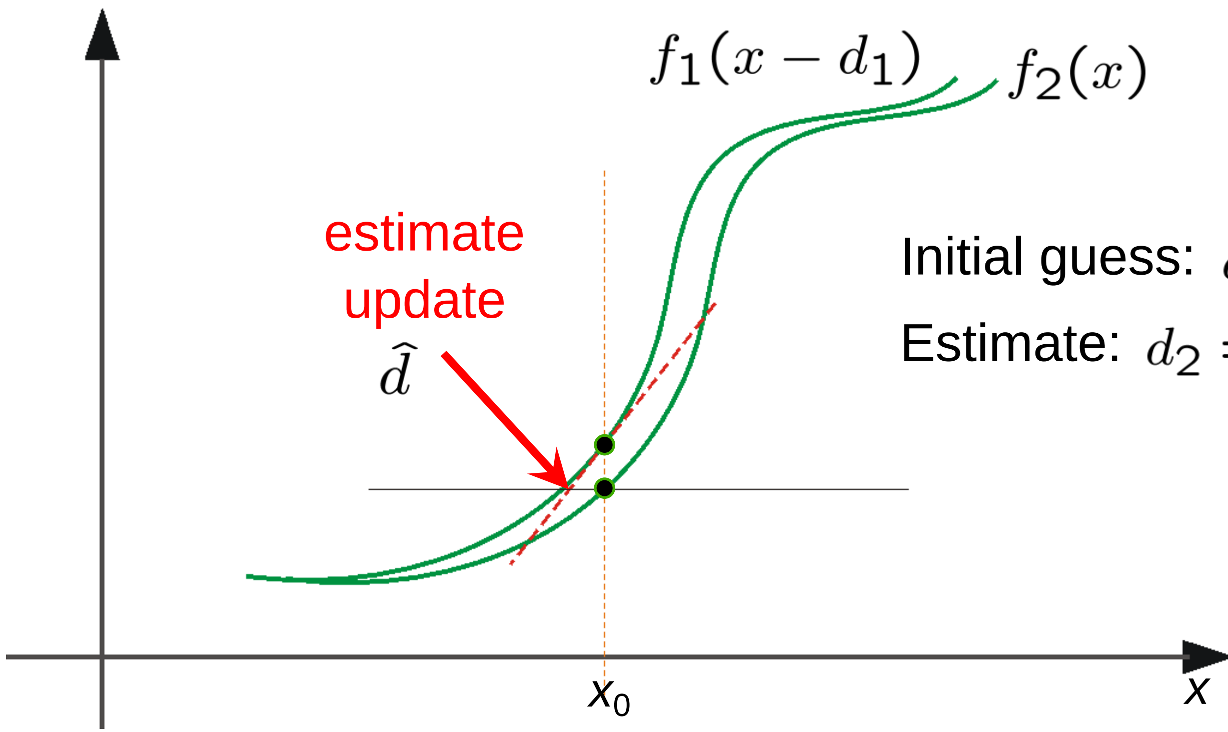


Initial guess: $d_0 = 0$

Estimate: $d_1 = d_0 + \hat{d}$

(using *d* for *displacement* here instead of *u*)

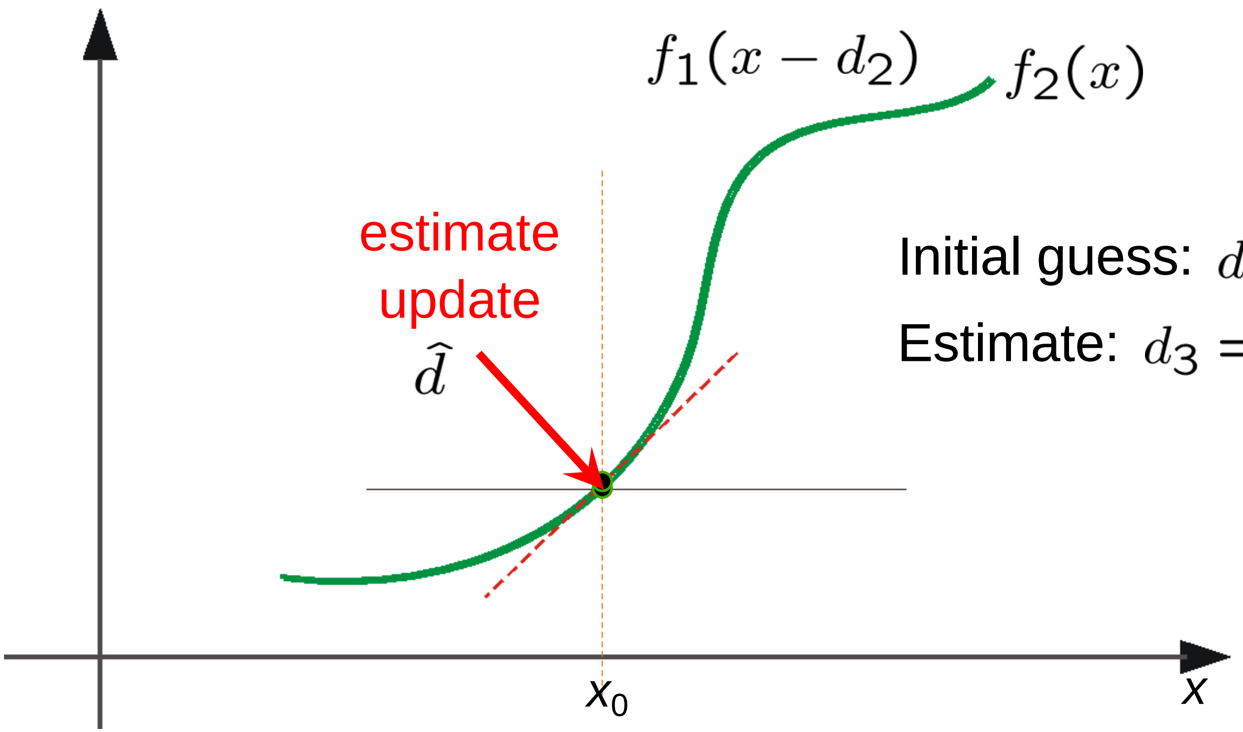
Optical Flow: Iterative Estimation



Initial guess: d_1

Estimate: $d_2 = d_1 + \hat{d}$

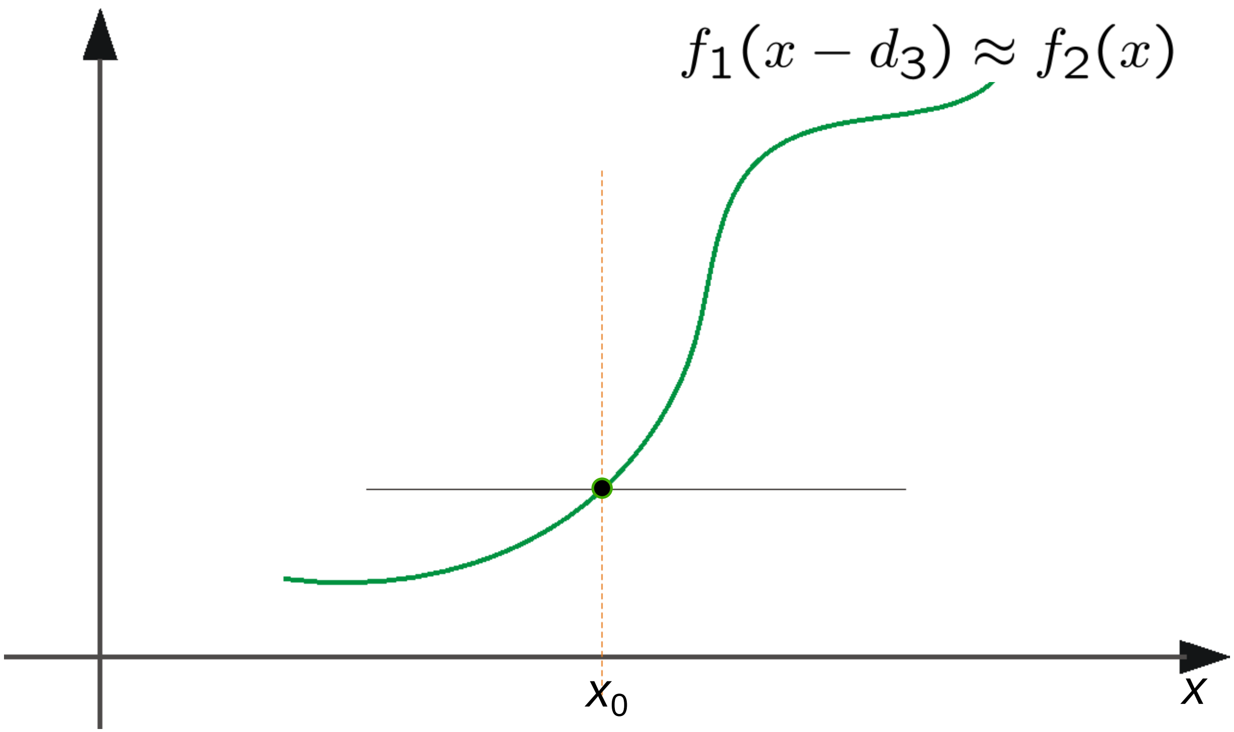
Optical Flow: Iterative Estimation



Initial guess: d_2

Estimate: $d_3 = d_2 + \hat{d}$

Optical Flow: Iterative Estimation



Optical Flow: Iterative Estimation

Some Implementation Issues:

- Warping is not easy (ensure that errors in warping are smaller than the estimate refinement)
- Warp one image, take derivatives of the other so you don't need to re-compute the gradient after each iteration.
- Often useful to low-pass filter the images before motion estimation (for better derivative estimation, and linear approximations to image intensity)

Revisiting the small motion assumption

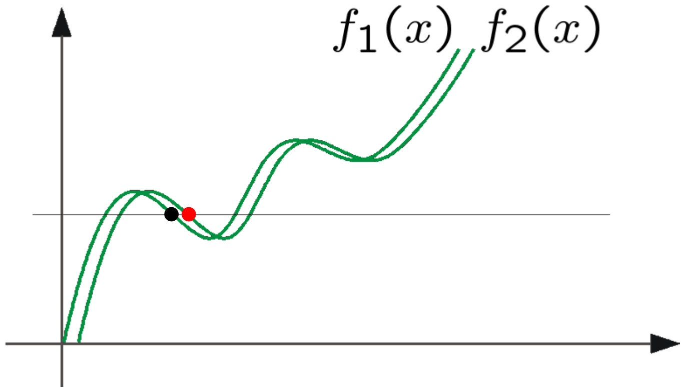
Is this motion small enough?

- Probably not—it's much larger than one pixel (2^{nd} order terms dominate)
- How might we solve this problem?

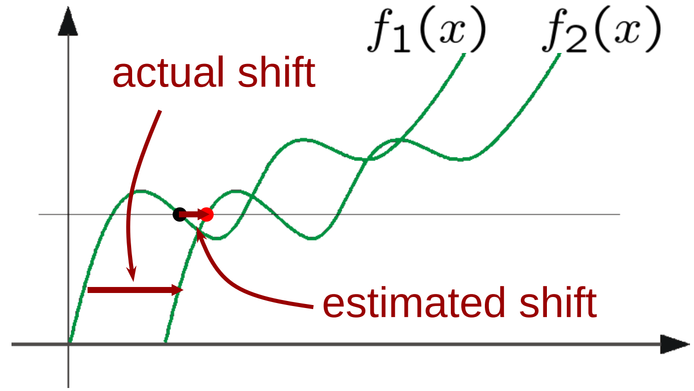
Optical Flow: Aliasing

Temporal aliasing causes ambiguities in optical flow because images can have many pixels with the same intensity.

I.e., how do we know which ‘correspondence’ is correct?



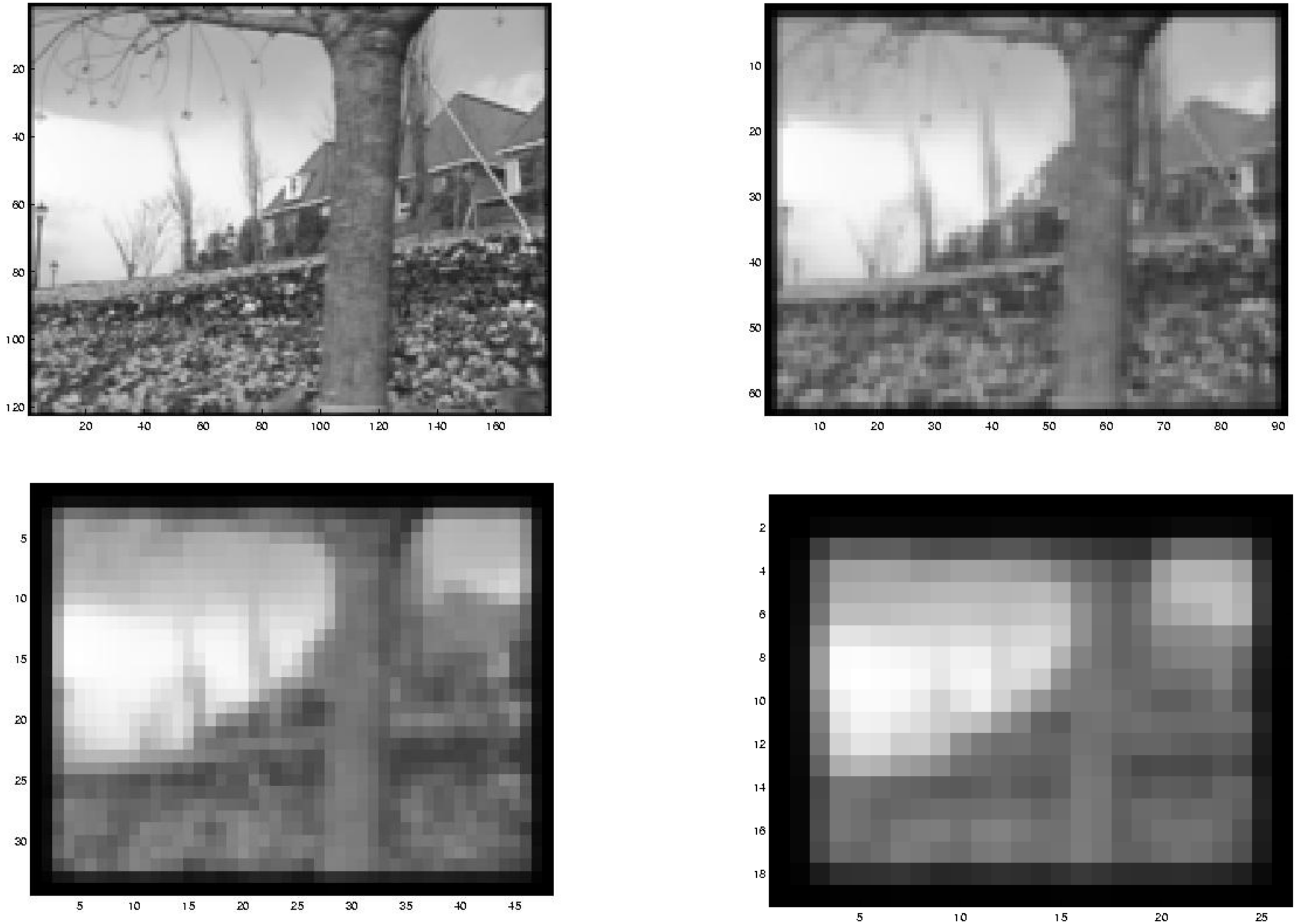
nearest match is correct (no aliasing)



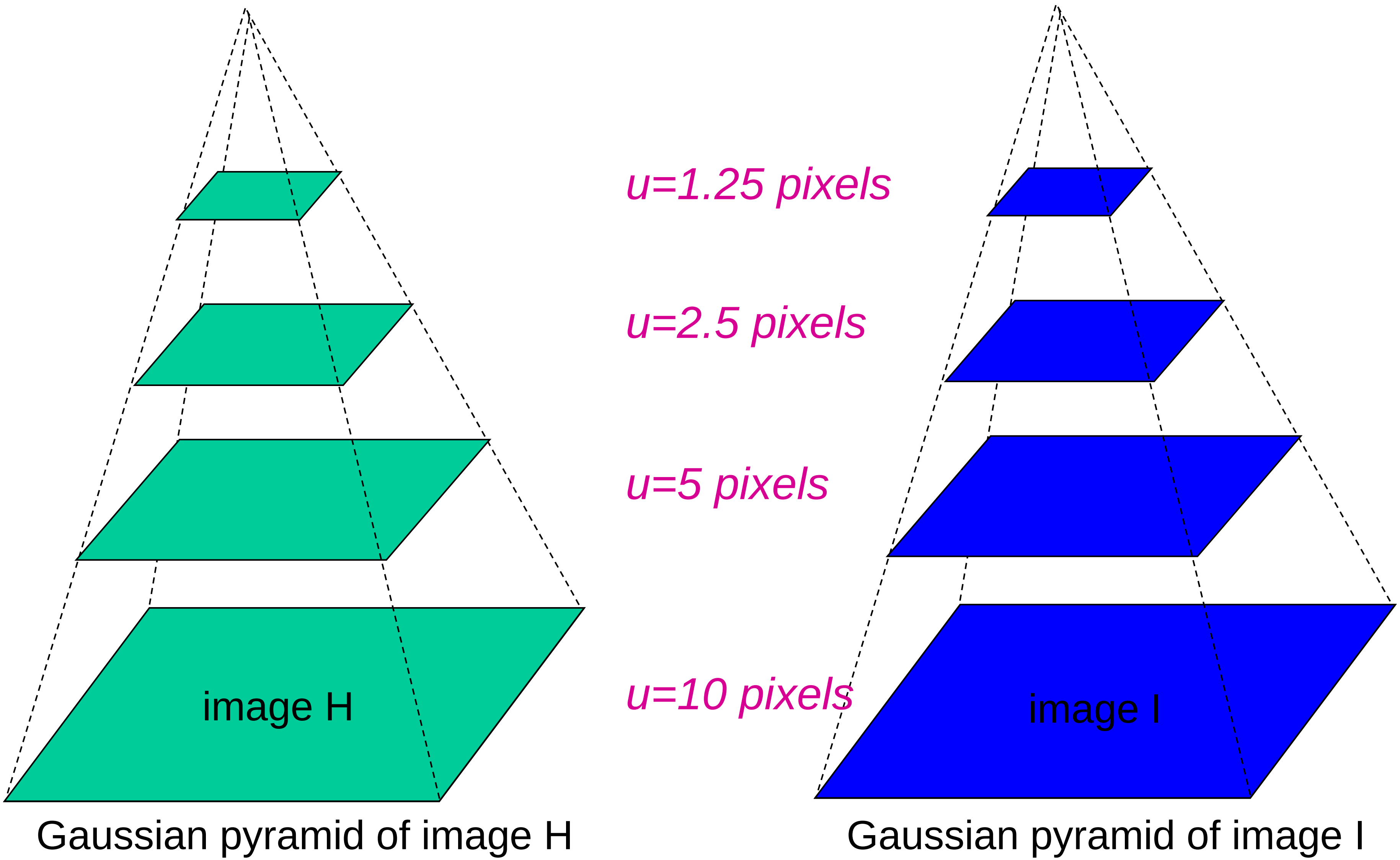
nearest match is incorrect (aliasing)

To overcome aliasing: coarse-to-fine estimation.

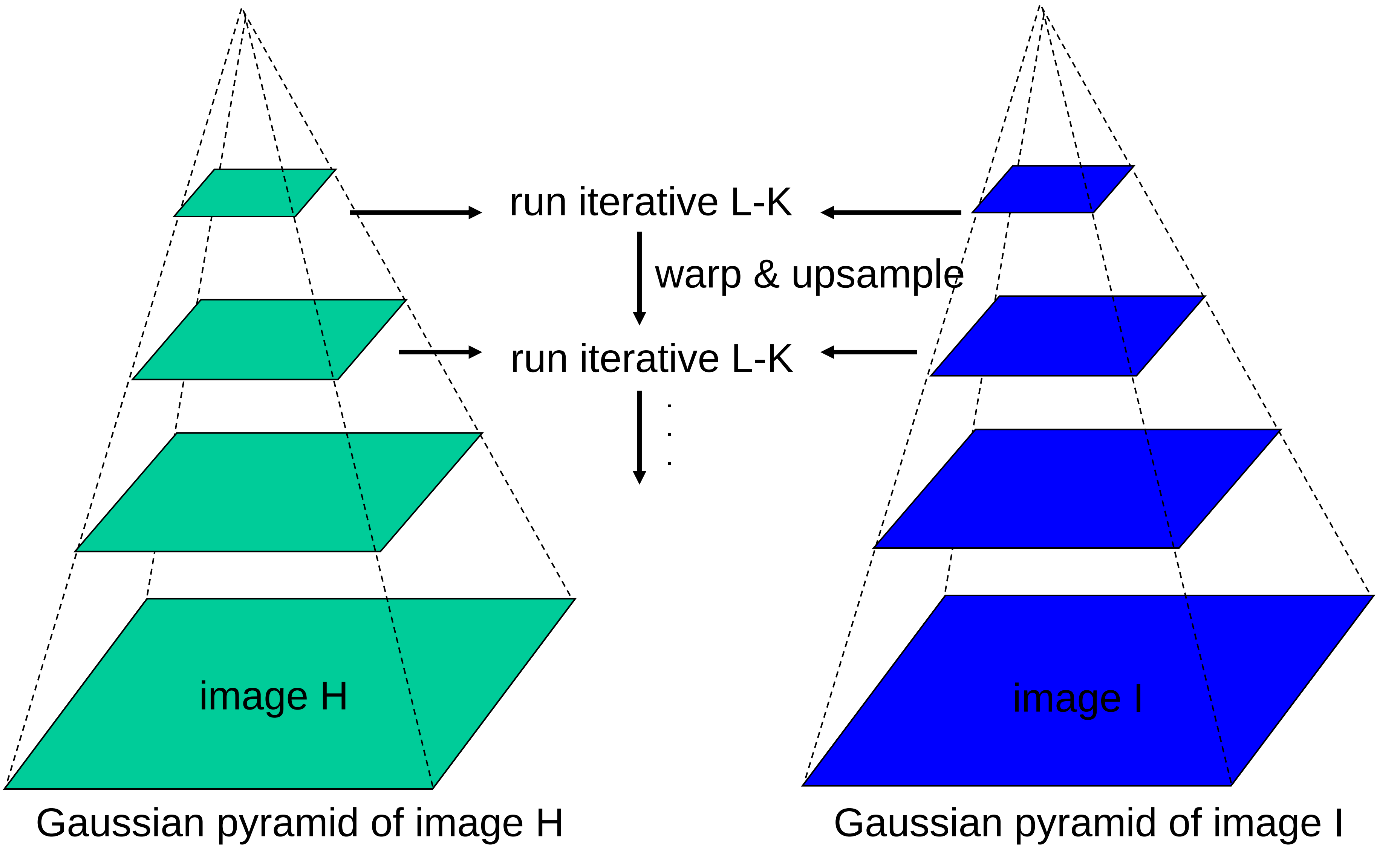
Reduce the resolution!



Coarse-to-fine optical flow estimation



Coarse-to-fine optical flow estimation



Beyond Translation

So far, our patch can only translate in (u,v)

What about other motion models?

- rotation, affine, perspective

Same thing but need to add an appropriate Jacobian

See Szeliski's survey of Panorama stitching

$$\mathbf{A}^{\mathbf{T}}\mathbf{A} = \sum_i \mathbf{J}\nabla\mathbf{I}(\nabla\mathbf{I})^{\mathbf{T}}\mathbf{J}^{\mathbf{T}}$$

$$\mathbf{A}^{\mathbf{T}}\mathbf{b} = -\sum_i \mathbf{J}^{\mathbf{T}} I_t (\nabla\mathbf{I})^{\mathbf{T}}$$

Recap: Classes of Techniques

Feature-based methods (e.g. SIFT+Ransac+regression)

- Extract visual features (corners, textured areas) and track them over multiple frames
- Sparse motion fields, but possibly robust tracking
- Suitable especially when image motion is large (10-s of pixels)

Direct-methods (e.g. optical flow)

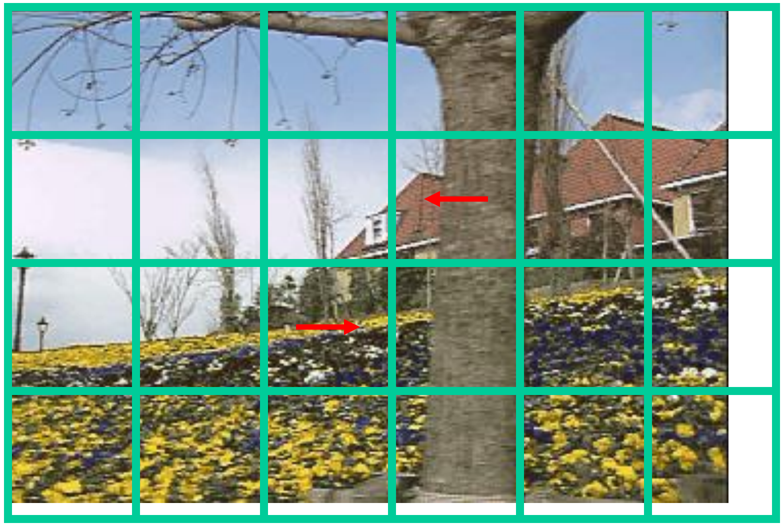
- Directly recover image motion from spatio-temporal image brightness variations
- Global motion parameters directly recovered without an intermediate feature motion calculation
- Dense motion fields, but more sensitive to appearance variations
- Suitable for video and when image motion is small (< 10 pixels)

Block-based motion prediction

Break image up into square blocks

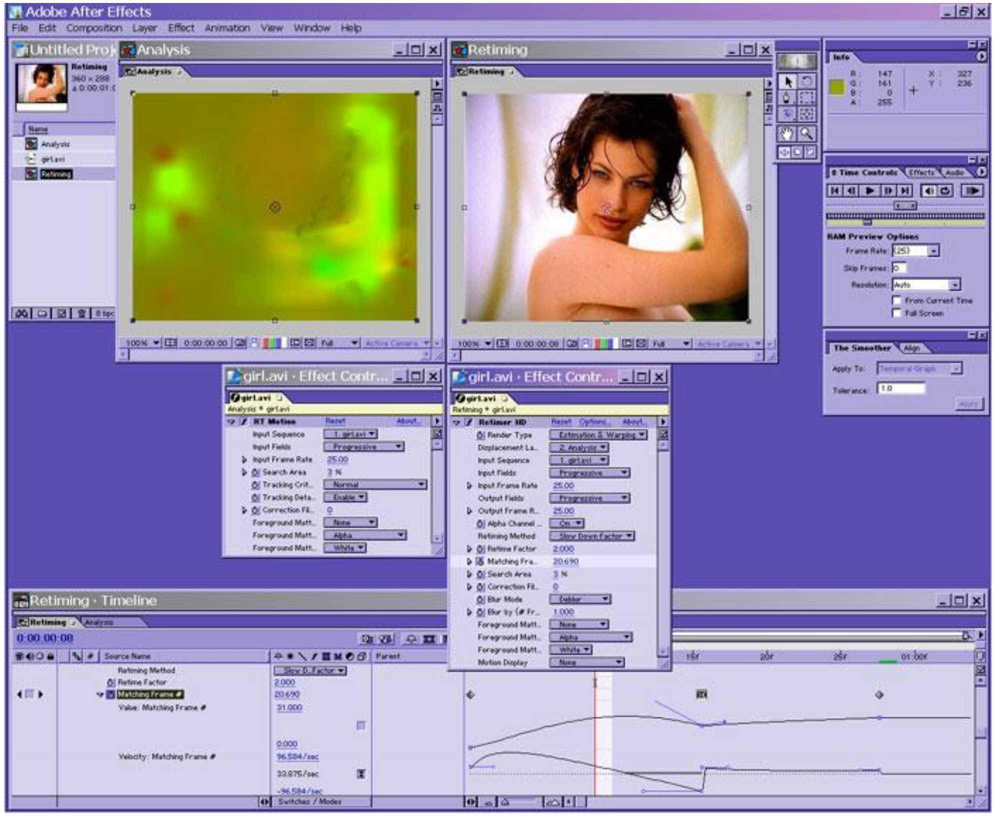
Estimate translation for each block

Use this to predict next frame, code difference (MPEG-2)



Retiming

http://www.realviz.com/retiming.htm



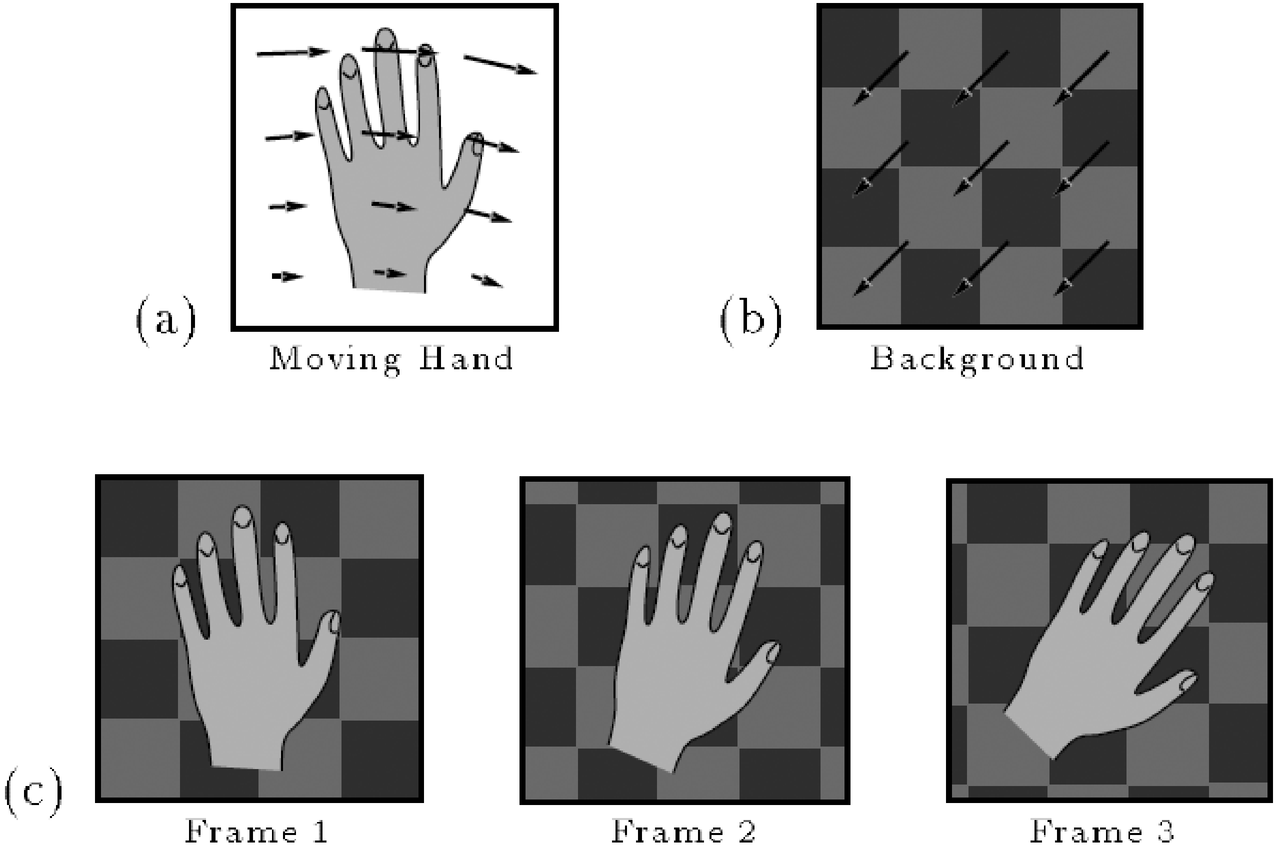
Layered motion

- Break image sequence into “layers” each of which has a coherent motion

J. Wang and E. Adelson. [Layered Representation for Motion Analysis.](Layered Representation for Motion Analysis) *CVPR 1993*.

What are layers?

- Each layer is defined by an alpha mask and an affine motion model



J. Wang and E. Adelson. [Layered Representation for Motion Analysis.](#) *CVPR 1993*.

Motion segmentation with an affine model

$$\boxed{u(x, y)} = a_1 + a_2 x + a_3 y$$
$$\boxed{v(x, y)} = a_4 + a_5 x + a_6 y$$

Local flow estimates

J. Wang and E. Adelson. [Layered Representation for Motion Analysis.](#) *CVPR 1993*.

Motion segmentation with an affine model

$$\boxed{u(x, y) = a_1 + a_2 x + a_3 y}$$
$$v(x, y) = a_4 + a_5 x + a_6 y$$

Equation of a plane (parameters a_1, a_2, a_3 can be found by least squares)

J. Wang and E. Adelson. [Layered Representation for Motion Analysis.](#) *CVPR 1993*.

Motion segmentation with an affine model

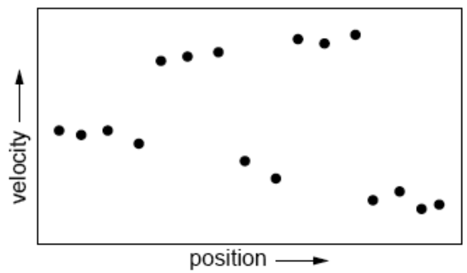
$$\boxed{u(x,y) = a_1 + a_2 x + a_3 y}$$

$$v(x,y) = a_4 + a_5 x + a_6 y$$

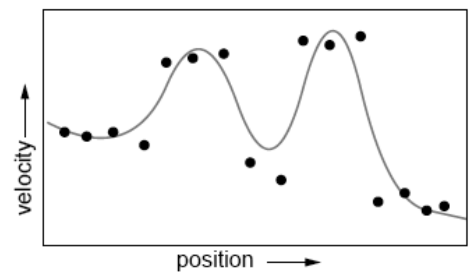
Equation of a plane (parameters a_1, a_2, a_3 can be found by least squares)

1D example

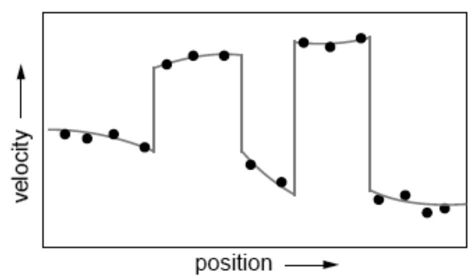
$u(x,y)$



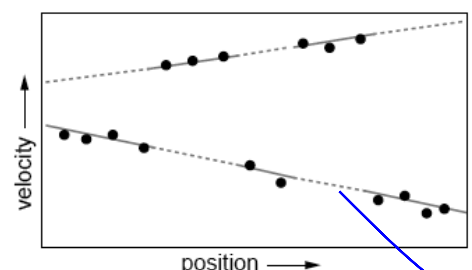
True flow



Local flow estimate



Segmented estimate



"Foreground"

"Background"

Occlusion

Line fitting

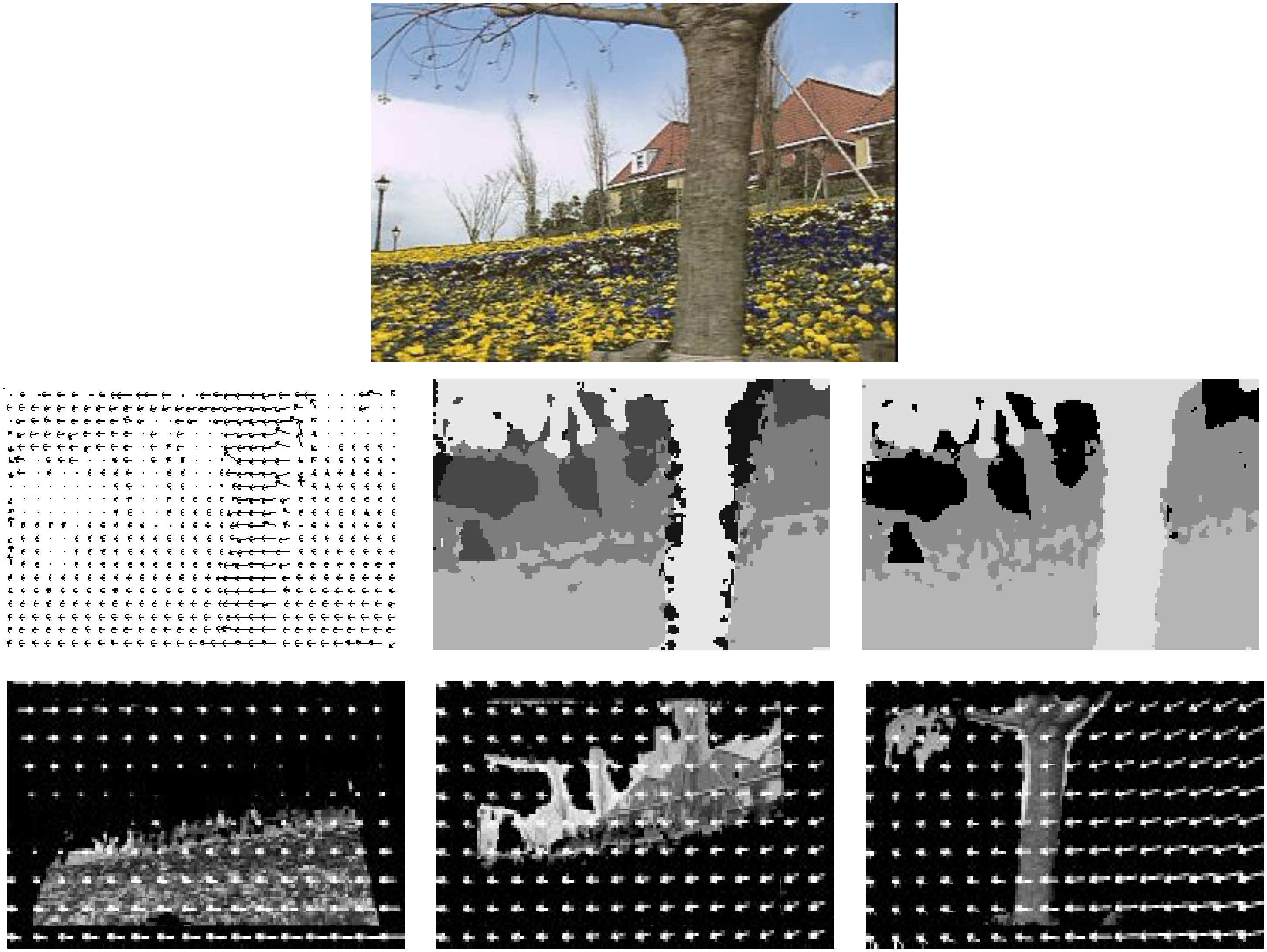
J. Wang and E. Adelson. [Layered Representation for Motion Analysis.](#) *CVPR 1993*.

How do we estimate the layers?

- Compute local flow in a coarse-to-fine fashion
- Obtain a set of initial affine motion hypotheses
  - Divide the image into blocks and estimate affine motion parameters in each block by least squares
    - Eliminate hypotheses with high residual error
  - Perform k-means clustering on affine motion parameters
    - Merge clusters that are close and retain the largest clusters to obtain a smaller set of hypotheses to describe all the motions in the scene
- Iterate until convergence:
  - Assign each pixel to best hypothesis
    - Pixels with high residual error remain unassigned
  - Perform region filtering to enforce spatial constraints
  - Re-estimate affine motions in each region

J. Wang and E. Adelson. Layered Representation for Motion Analysis. *CVPR 1993*.

Example result



J. Wang and E. Adelson. [Layered Representation for Motion Analysis.](#) *CVPR 1993*.

Overview

- Segmentation in Video
- Optical flow
- Motion Magnification

Motion Magnification

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Frédo Durand Edward H. Adelson

Computer Science and Artificial Intelligence Laboratory
Massachusetts Institute of Technology

Motion Microscopy

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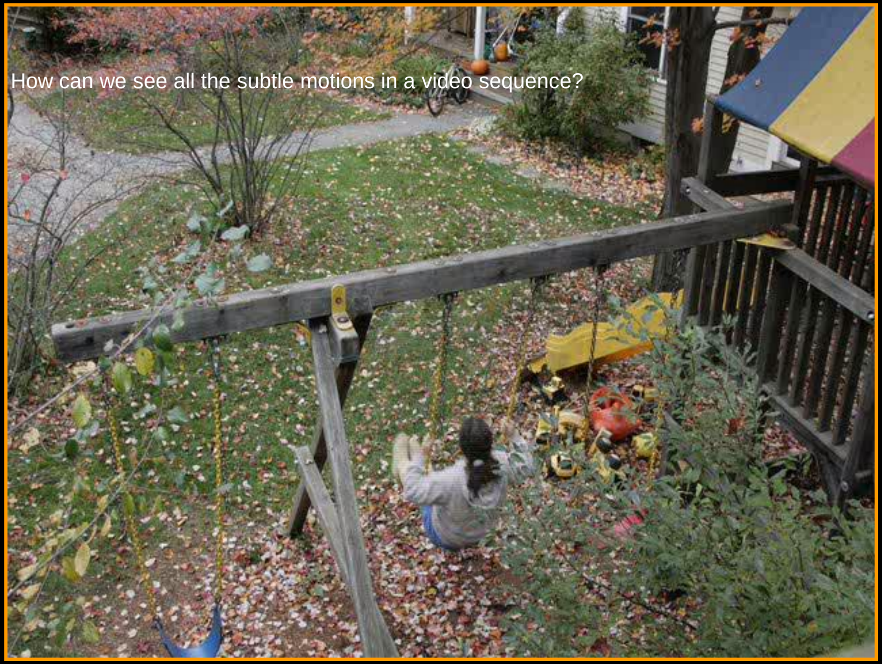

Original sequence


Magnified sequence

Naïve Approach

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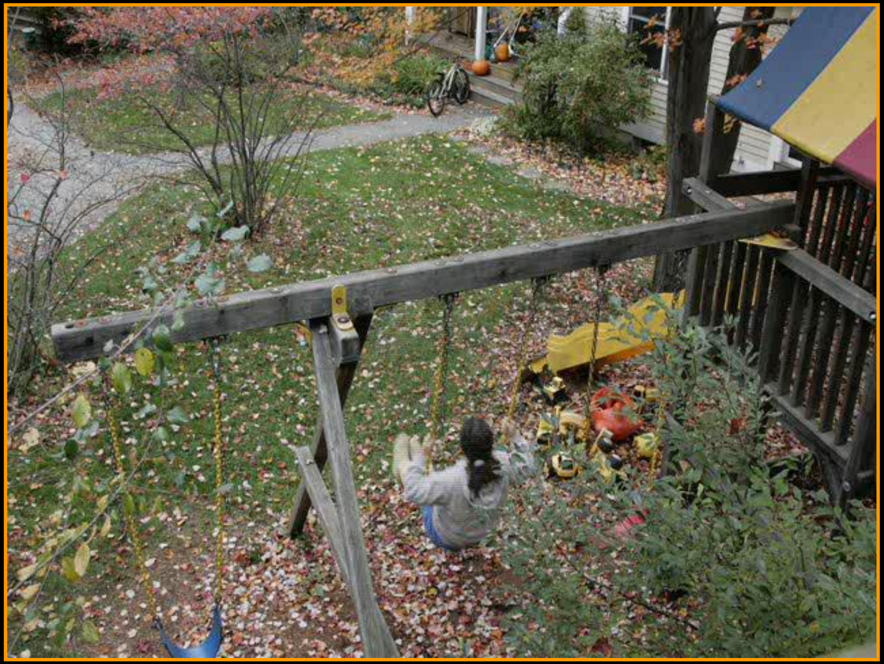
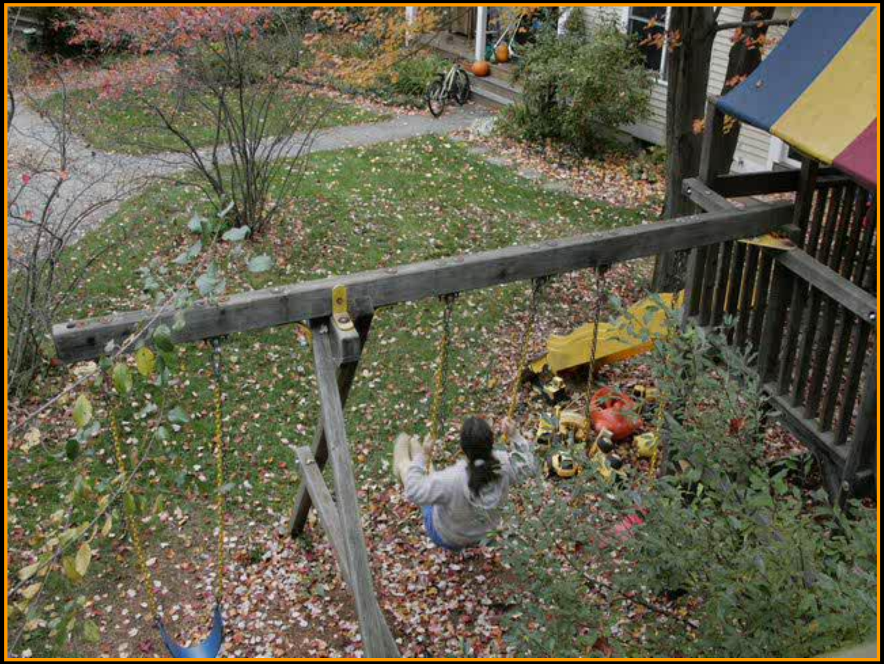
- Magnify the estimated optical flow field
- Rendering by warping



Original sequence



Magnified by naïve approach

Layer-based Motion Magnification Processing Pipeline

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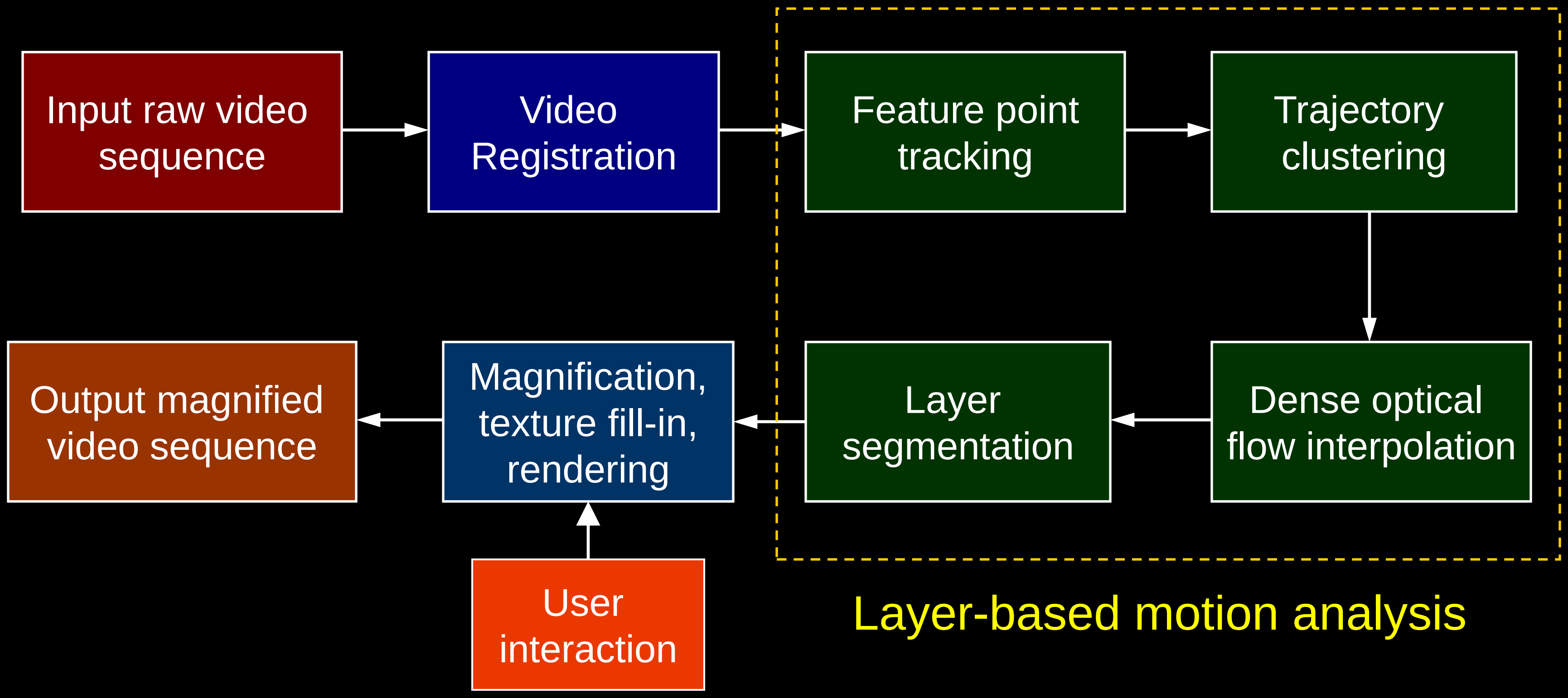
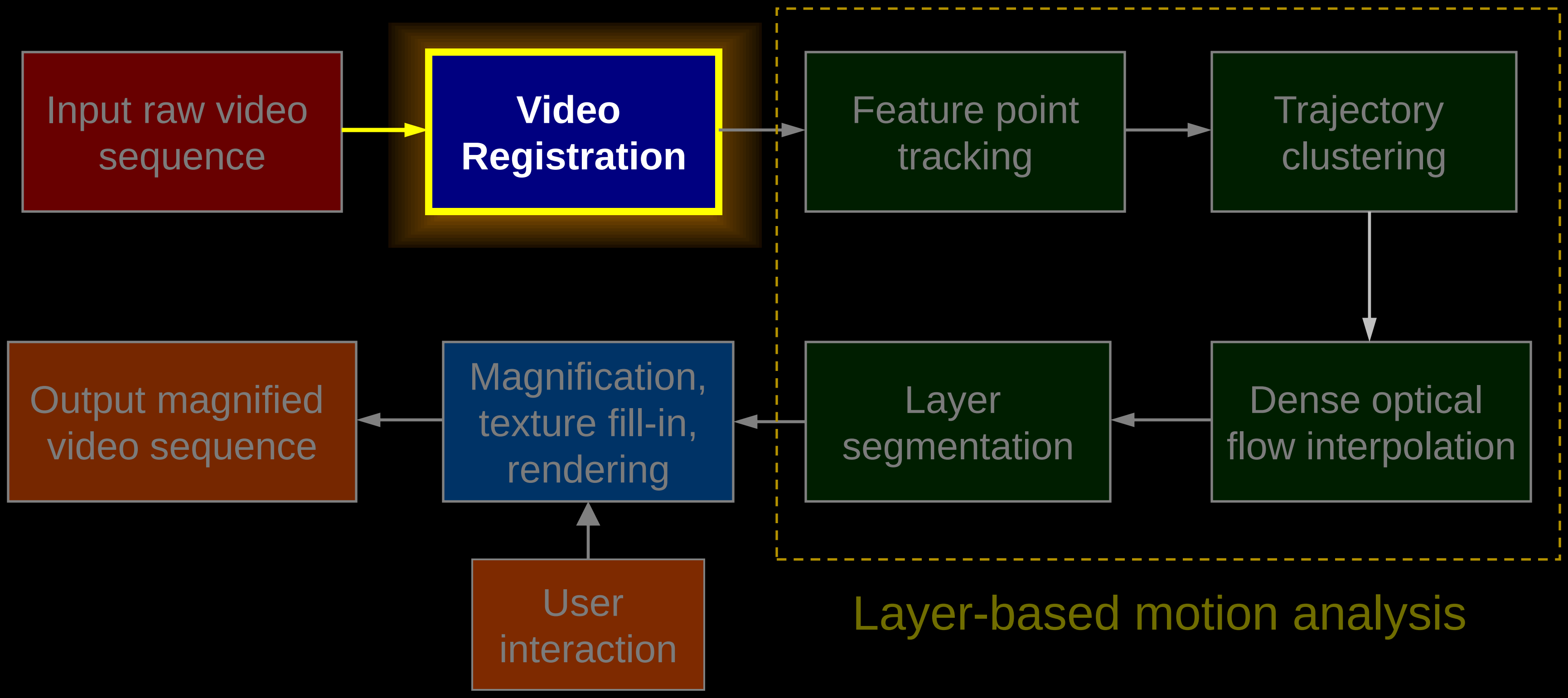


Stationary camera, stationary background

Layer-based Motion Magnification

Video Registration



Stationary camera, stationary background

Robust Video Registration

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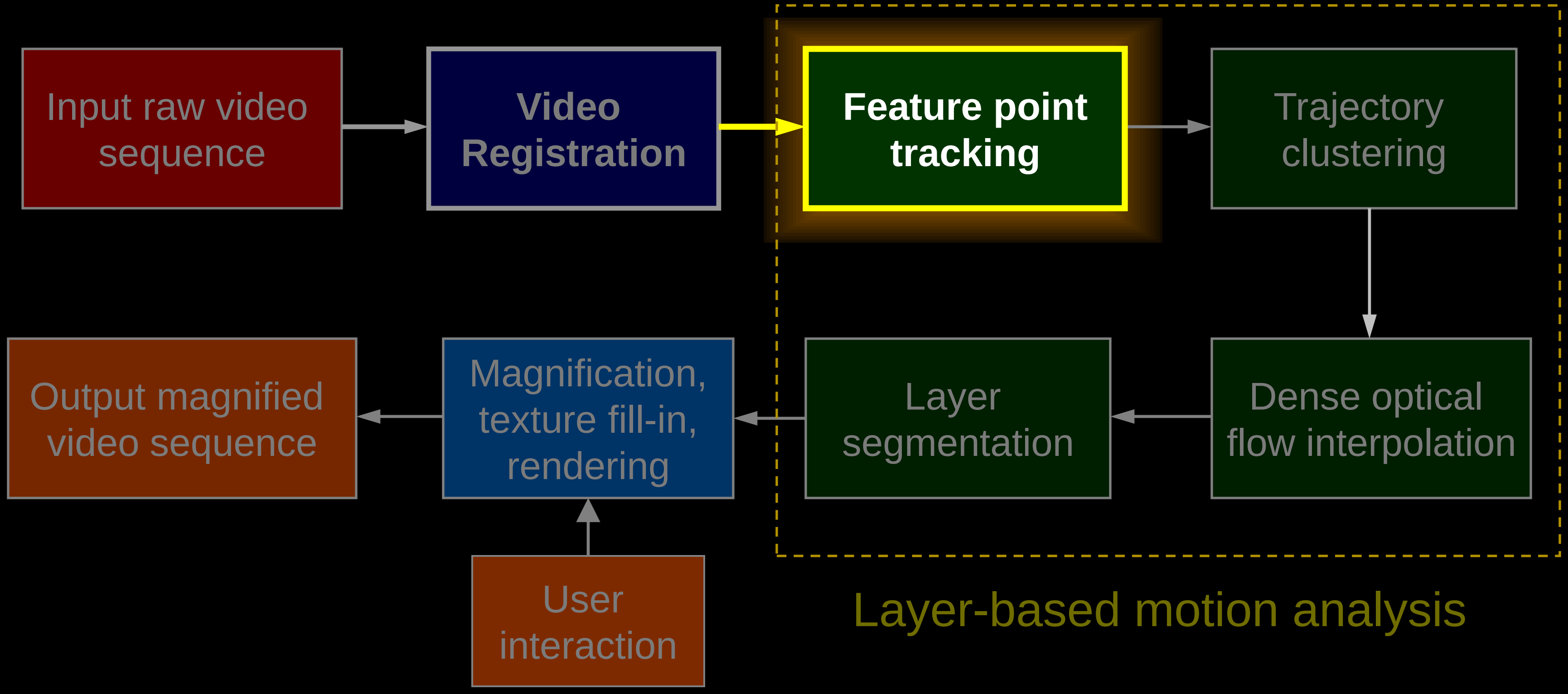
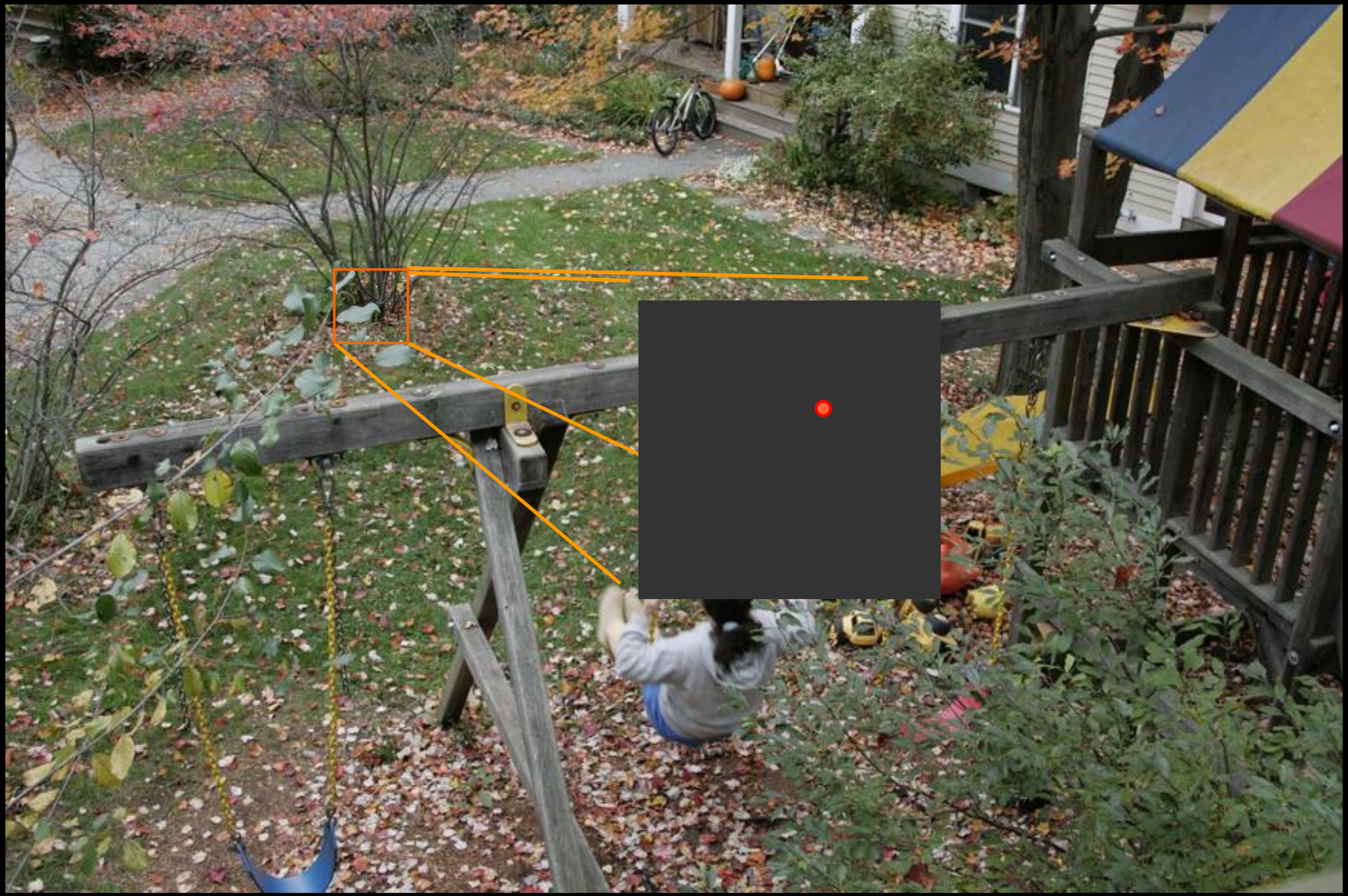
- Find feature points with Harris corner detector on the reference frame
- Brute force tracking feature points
- Select a set of robust feature points with inlier and outlier estimation (most from the rigid background)
- Warp each frame to the reference frame with a global affine transform

Motion Magnification Pipeline
Feature Point Tracking



Challenges (1)



Adaptive Region of Support

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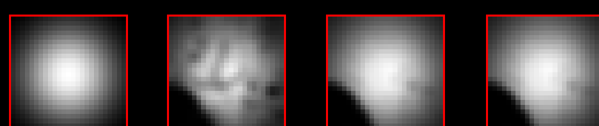
- Brute force search

Confused by occlusion !

time

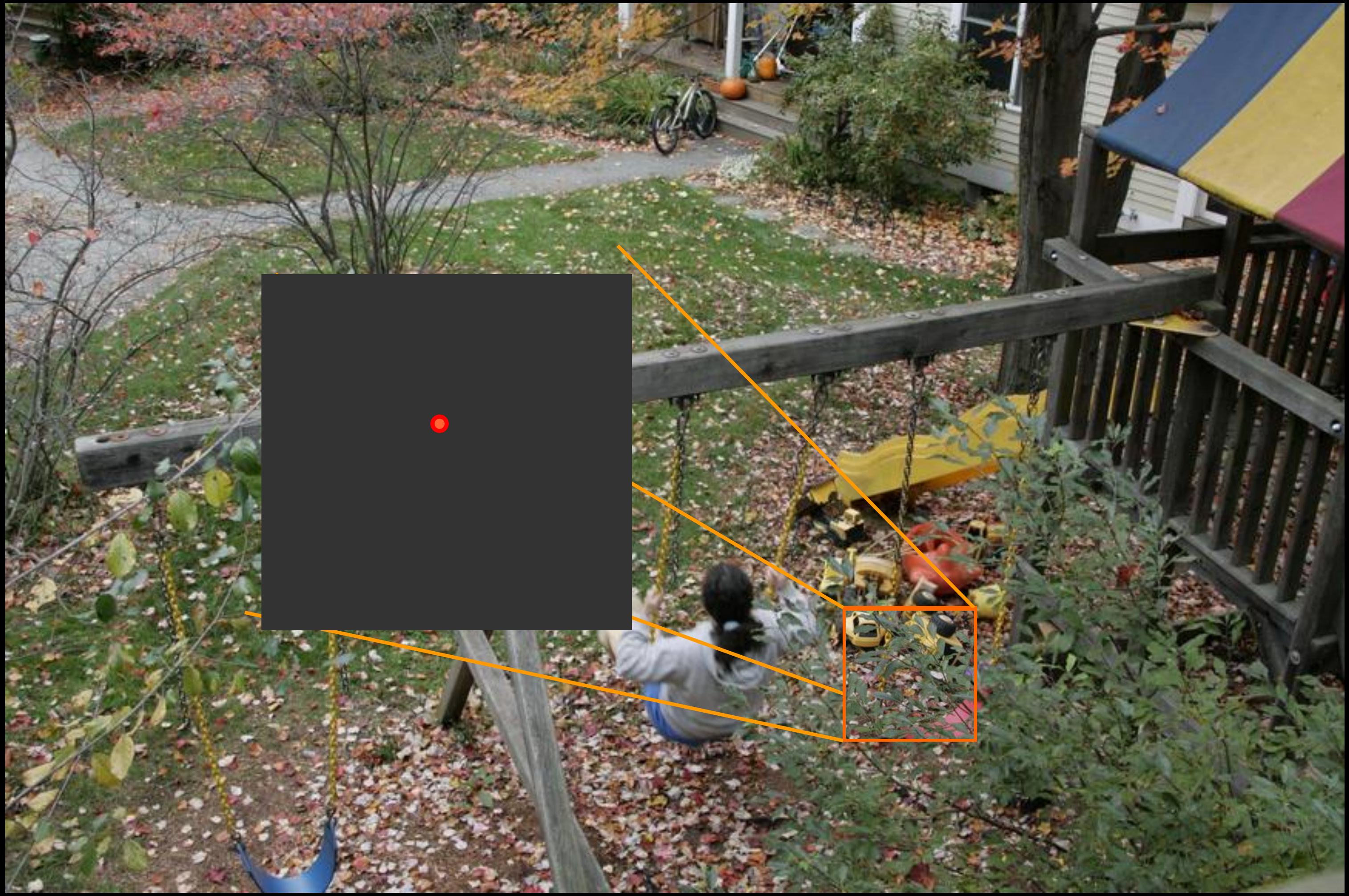
- Learn adaptive region of support using expectation-maximization (EM) algorithm

region of support



time

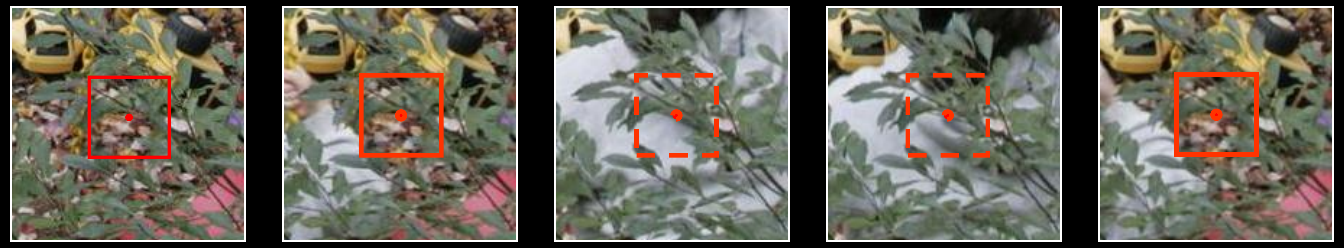
Challenges (2)



Trajectory Pruning

- Tracking with adaptive region of support Nonsense at full occlusion!

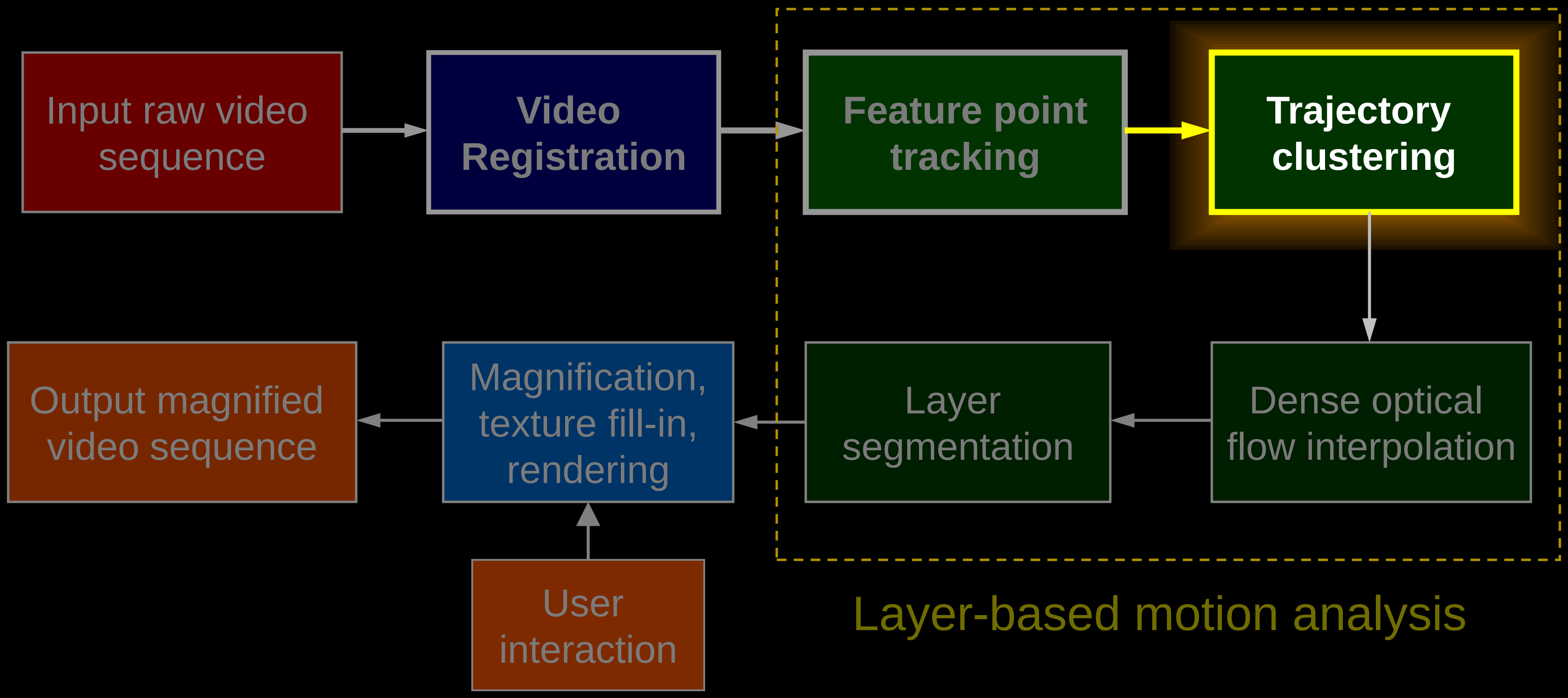
- Outlier detection and removal by interpolation



Comparison

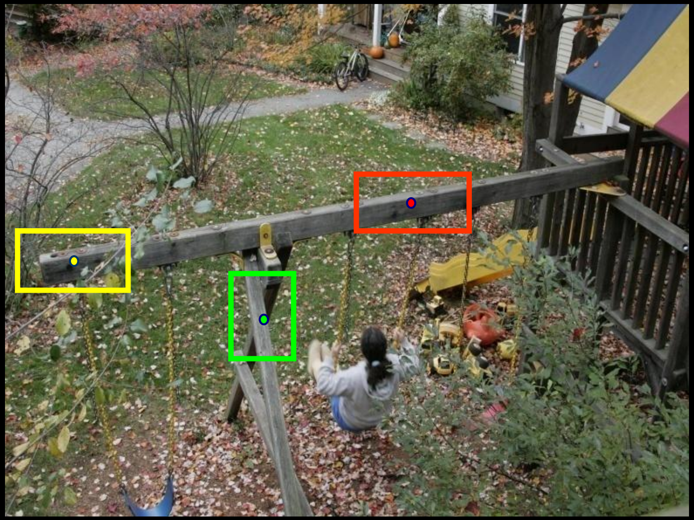
Without With adaptive region of support and trajectory pruning

Motion Magnification Pipeline
Trajectory Clustering



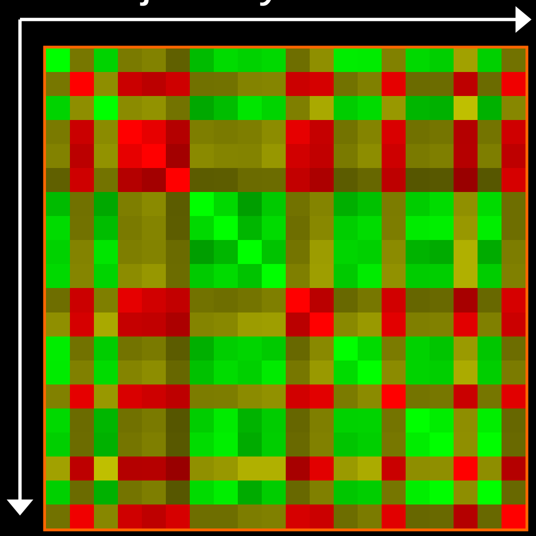
Normalized Complex Correlation

- The similarity metric should be independent of phase and magnitude
- Normalized complex correlation



$$S(C_1, C_2) = \frac{\left| \sum_t C_1(t)\overline{C}_2(t) \right|^2}{\sqrt{\sum_t C_1(t)\overline{C}_1(t)}\sqrt{\sum_t C_2(t)\overline{C}_2(t)}}$$

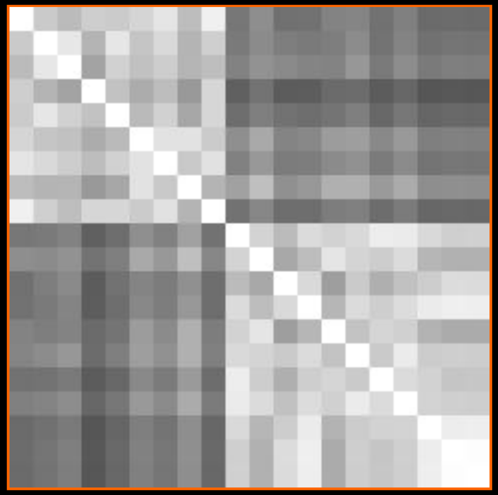
Spectral Clustering



Affinity matrix

Two clusters

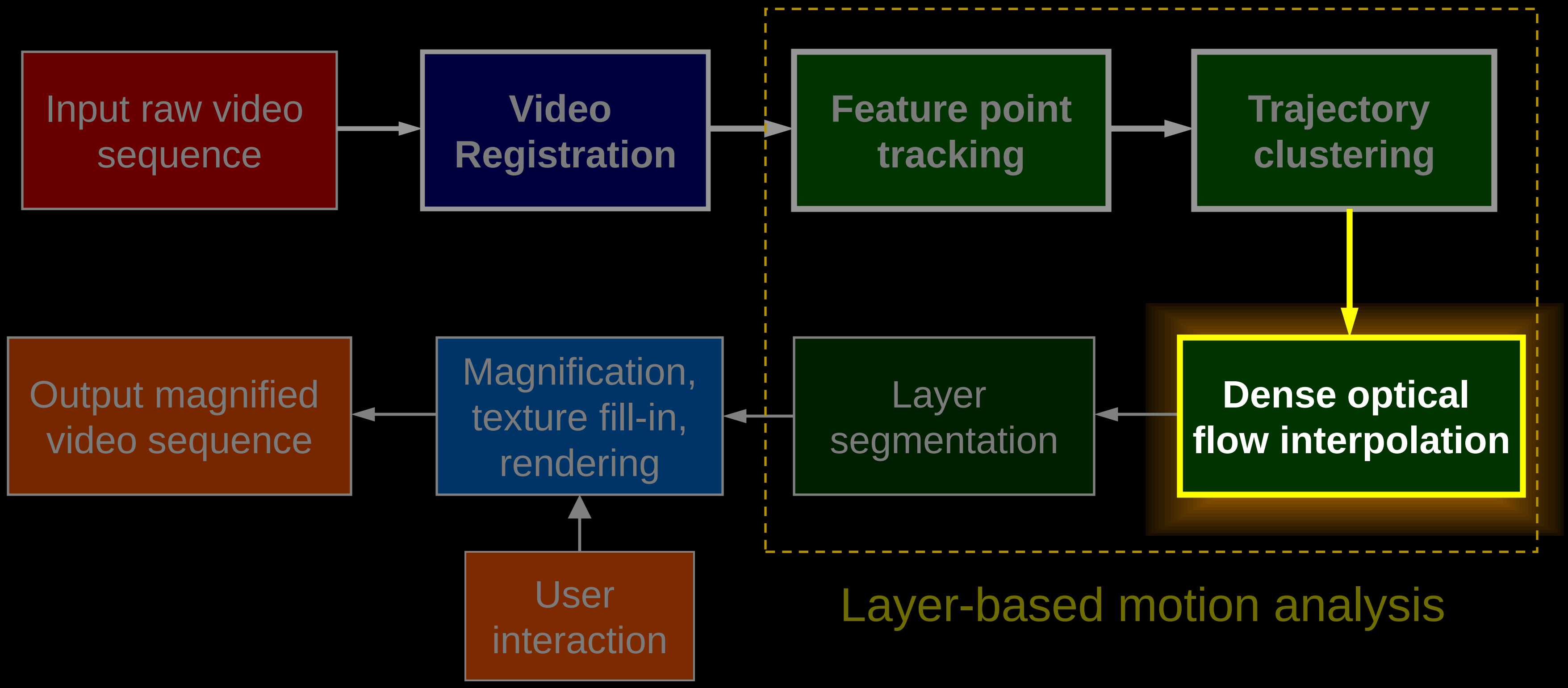
Clustering



Reordering of affinity matrix

Clustering Results

Motion Magnification Pipeline
Dense Optical Flow Field



From Sparse Feature Points to Dense Optical Flow Field

- Interpolate dense optical flow field using locally weighted linear regression

Flow vectors of clustered sparse feature points

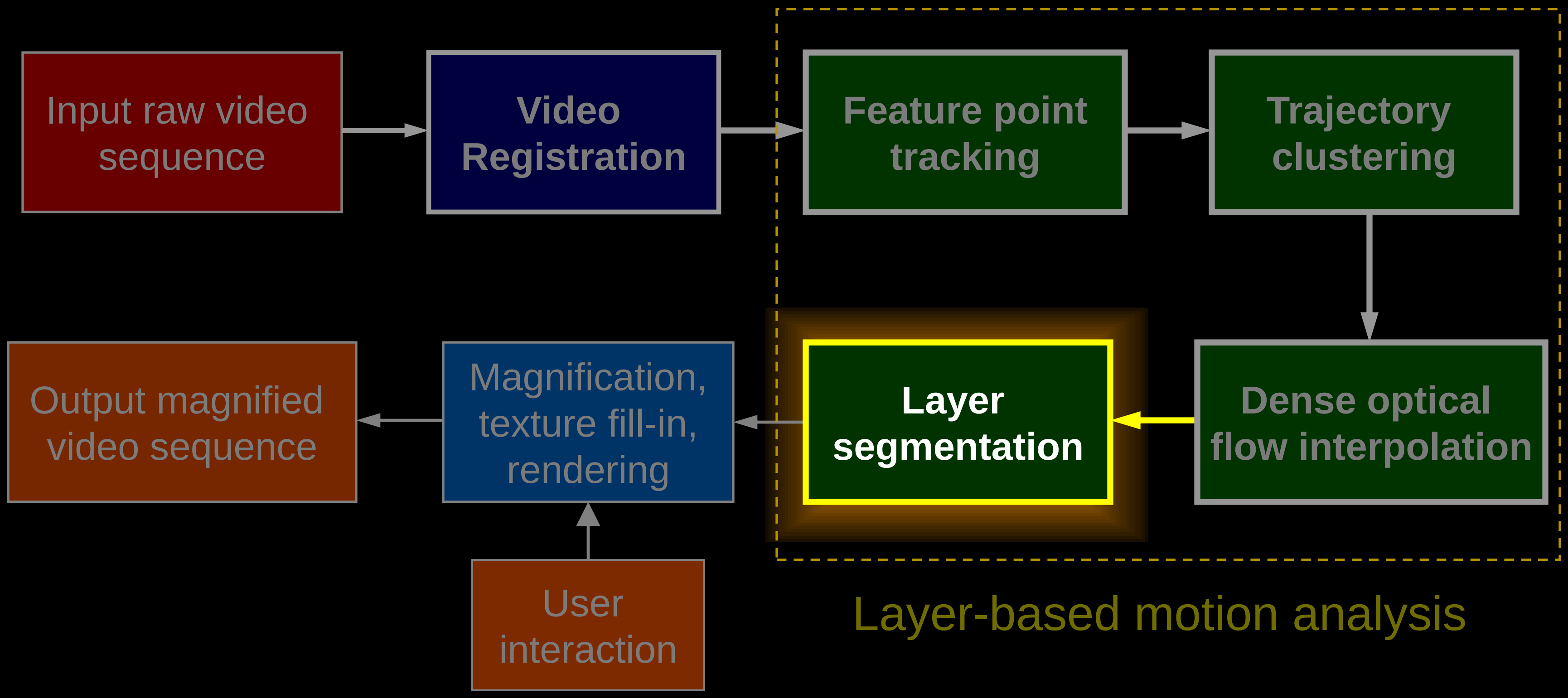
Dense optical flow field of cluster 2 (swing)

(leaves)

Cluster 1: leaves

Cluster 2: swing

Motion Magnification Pipeline
Layer Segmentation



Motion Layer Assignment

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- Assign each pixel to a motion cluster layer, using four cues:
  - **Motion likelihood**—consistency of pixel's intensity if it moves with the motion of a given layer (dense optical flow field)
  - **Color likelihood**—consistency of the color in a layer
  - **Spatial connectivity**—adjacent pixels favored to belong the same group
  - **Temporal coherence**—label assignment stays constant over time
- Energy minimization using graph cuts

Segmentation Results

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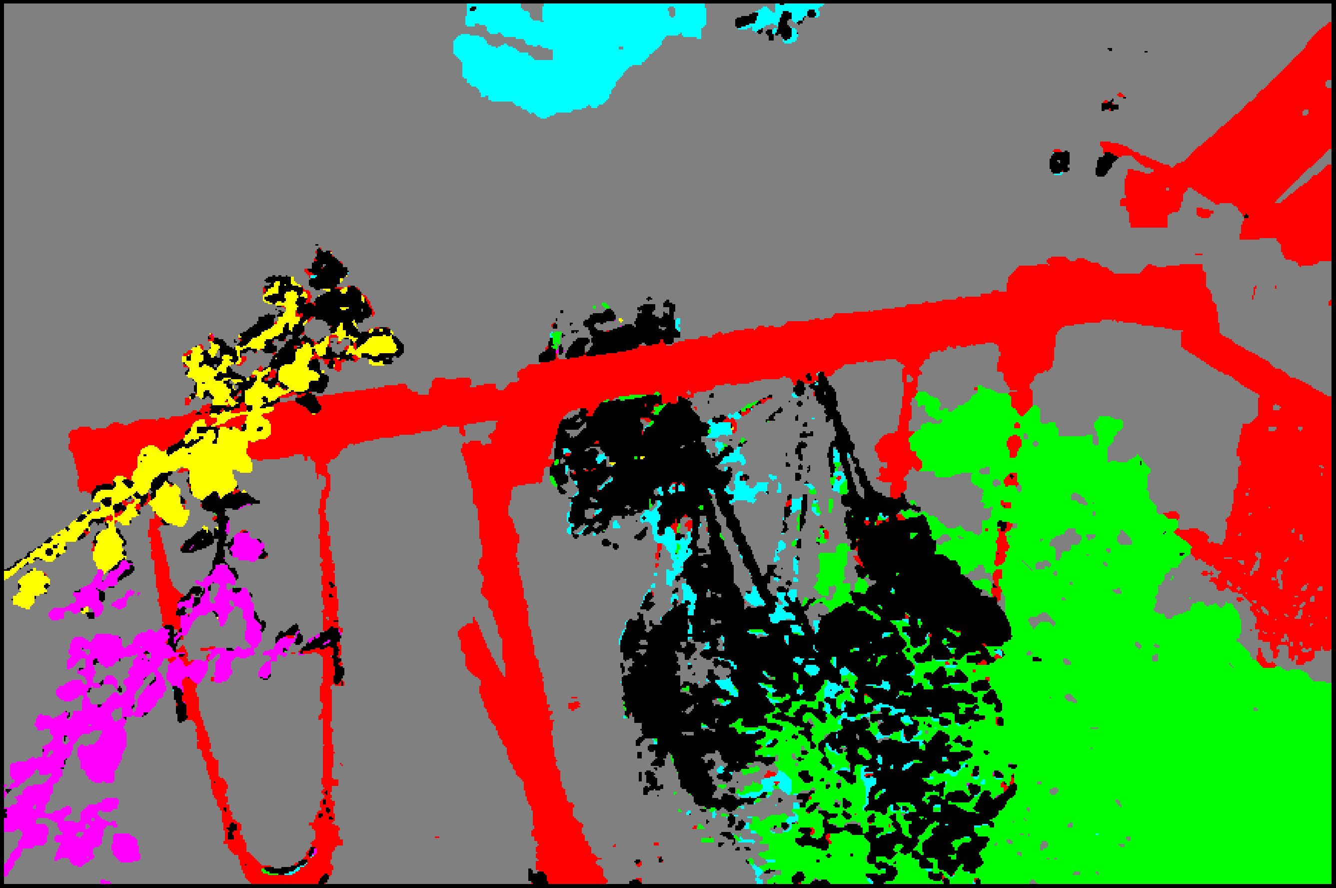
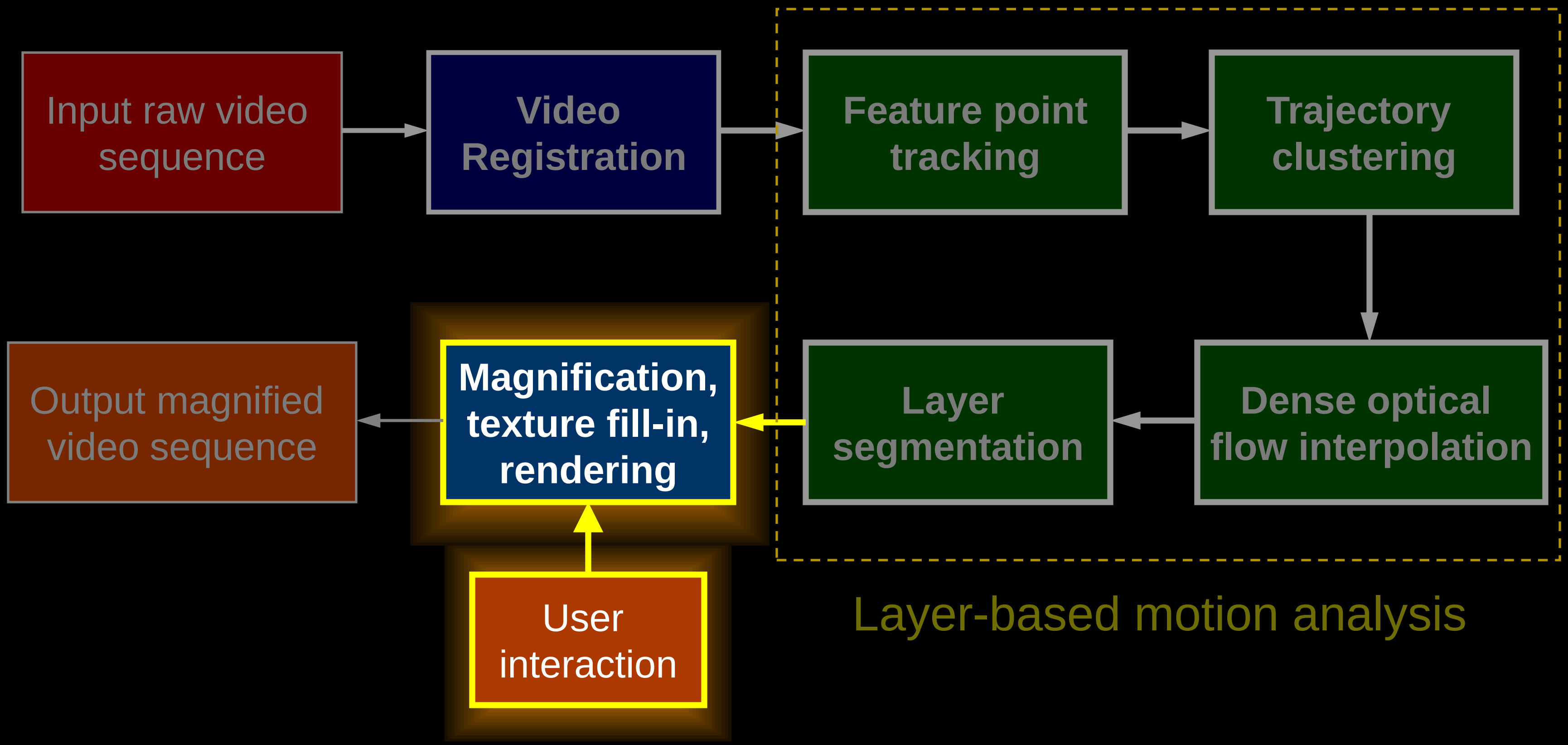
- Two additional layers: static background and outlier



Motion Magnification Pipeline
Editing and Rendering



Layered Motion Representation for Motion Processing

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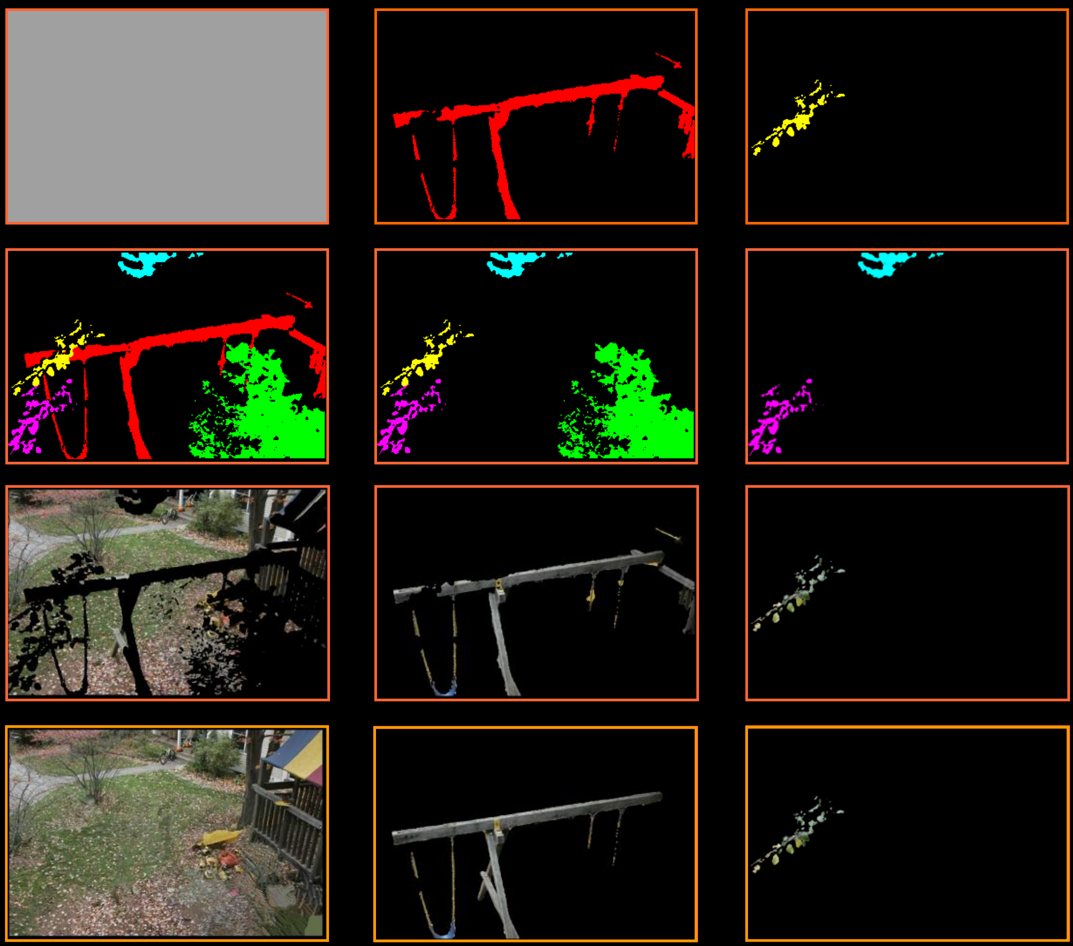
Background	Layer 1	Layer 2	



Layer mask

Occluding layers

Appearance for each layer before texture filling-in

Appearance for each layer after texture filling-in

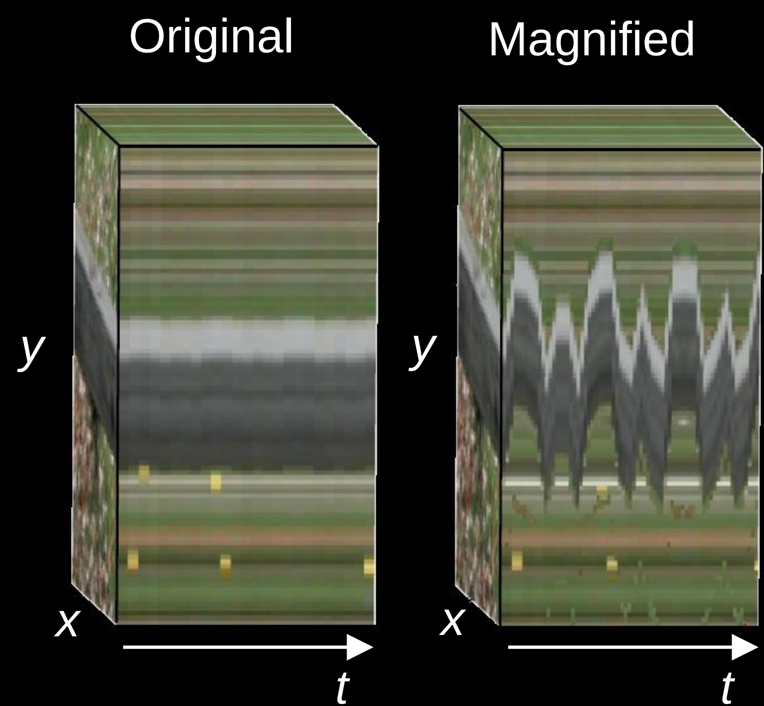
Appearance for each layer after user editing

Video

Motion Magnification

Is the Baby Breathing?

Are the Motions Real?



Are the Motions Real?

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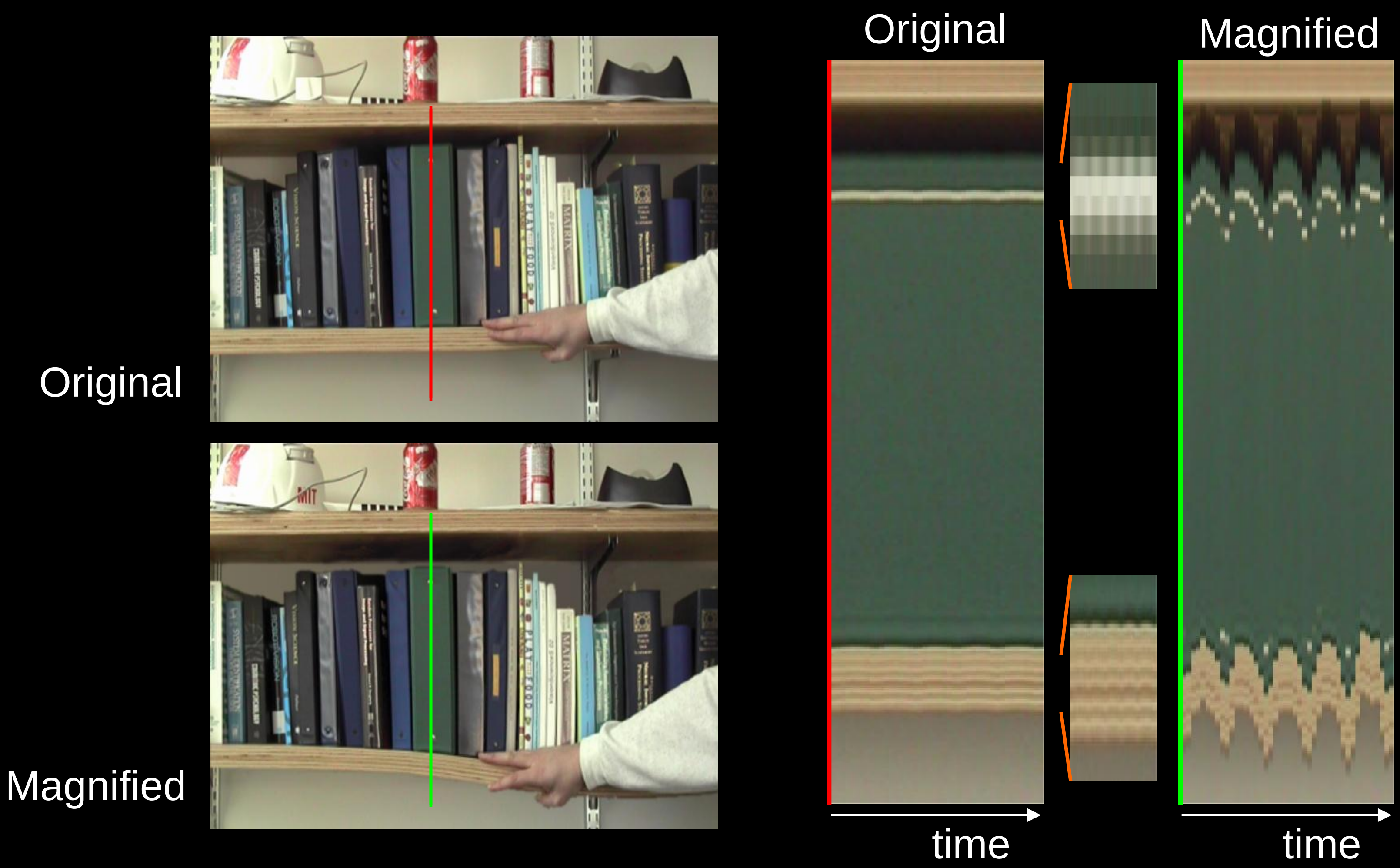

Applications

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- Education
- Entertainment
- Mechanical engineering
- Medical diagnosis

Conclusion

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- Motion magnification
  - A motion microscopy technique
- Layer-based motion processing system
  - Robust feature point tracking
  - Reliable trajectory clustering
  - Dense optical flow field interpolation
  - Layer segmentation combining multiple cues

Thank you!

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Motion Magnification

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